Answer on Question #63336 – Math – Calculus

Question

To fill an order for 100 units of a product, a firm wishes to distribute the production between its two plants, Plant 1 and Plant 2. The total cost function is given by

c=f(q_1,q_2)=0.5q_1^2+2q_1+32q_2+500

where q_1 and q_2 are the number of units produced at Plants 1 and 2, respectively. How should the output be distributed in order to minimize costs?

Solution

q_1 – number of units produced at Plant 1,
q_2 – number of units produced at Plant 2.

$q_1 + q_2 = 100$ (units of a product).
$c = f(q_1, q_2) = 0.5q_1^2 + 2q_1 + 32q_2 + 500 \to$ min.
Obviously
$q_2 = 100 - q_1$.
We search for a minimum of function:

$$c(q_1, q_2) = 0.5q_1^2 + 2q_1 + 32(100 - q_1) + 500 = 0.5q_1^2 - 30q_1 + 3700.$$

Find the derivative:

$$\frac{d}{dq_1}(0.5q_1^2 - 30q_1 + 3700) = 0.5 \cdot 2q_1 - 30 = q_1 - 30 = 0 \Rightarrow q_1 = 30.$$

Find the second derivative:

$$\frac{d^2}{dq_1^2}(0.5q_1^2 - 30q_1 + 3700) = \frac{d}{dq_1}(q_1 - 30) = 1 > 0$$

=> point $q_1 = 30$ is minimum, hence $q_2 = 100 - q_1 = 100 - 30 = 70$.
To minimize costs, set $c(q_1, q_2) = c(30, 70)$.
Answer: $q_1 = 30$, $q_2 = 70$.