

Answer on Question #63324 – Math – Discrete Mathematics

Question

Using laws of logics how to show that

$$(\sim p \wedge (\sim q \wedge r)) \vee (q \wedge r) \vee (p \wedge r) \leftrightarrow r.$$

Solution

$$\begin{aligned} & (\sim p \wedge (\sim q \wedge r)) \vee (q \wedge r) \vee (p \wedge r) \Leftrightarrow && \text{Associative Law of disjunction} \\ \Leftrightarrow & (\sim p \wedge (\sim q \wedge r)) \vee ((q \wedge r) \vee (p \wedge r)) \Leftrightarrow && \text{Commutative Law of conjunction} \\ \Leftrightarrow & (\sim p \wedge (\sim q \wedge r)) \vee ((r \wedge q) \vee (r \wedge p)) \Leftrightarrow && \text{Distributive Law of conjunction over disjunction} \\ \\ \Leftrightarrow & (\sim p \wedge (\sim q \wedge r)) \vee (r \wedge (q \vee p)) \Leftrightarrow && \text{Associative Law of conjunction} \\ \Leftrightarrow & ((\sim p \wedge \sim q) \wedge r) \vee (r \wedge (q \vee p)) \Leftrightarrow && \text{Commutative Law of conjunction} \\ \Leftrightarrow & ((\sim p \wedge \sim q) \wedge r) \vee ((q \vee p) \wedge r) \Leftrightarrow && \text{DeMorgan's Law} \\ \Leftrightarrow & (\sim (p \vee q) \wedge r) \vee ((q \vee p) \wedge r) \Leftrightarrow && \text{Commutative Law of disjunction} \\ \Leftrightarrow & (\sim (p \vee q) \wedge r) \vee ((p \vee q) \wedge r) \Leftrightarrow && \text{Commutative Law of conjunction} \\ \Leftrightarrow & (r \wedge \sim (p \vee q)) \vee (r \wedge (p \vee q)) \Leftrightarrow && \\ \\ \Leftrightarrow & r \wedge (\sim (p \vee q) \vee (p \vee q)) \Leftrightarrow && \text{Distributive Law of conjunction over disjunction} \\ \Leftrightarrow & r \wedge T \Leftrightarrow && \text{Law of excluded middle} \\ \Leftrightarrow & r && \text{Redundance Law} \end{aligned}$$