Answer on Question #63157 - Math - Statistics and Probability

Question

Shell lengths of sea turtles. Refer to the Aquatic Biology (Vol. 9, 2010) study of green sea turtles inhabiting the Grand Cayman South Sound Iagoon, Exercise 2.83 (p. 65). Researchers discovered that the curved carapace (shell) length of these turtles is approximately normally distributed with mean 55.7 centimeters and standard deviation 11.5 centimeters.

- **a.** The minimum and maximum size limits for captured turtles in the legal marine turtle fishery are 40 cm and 60 cm, respectively. How likely are you to capture a green sea turtle that is considered illegal?
- **b.** What maximum limit, L, should be set so that only 10% of the turtles captured have shell lengths greater than L?

Solution

a.
$$z = \frac{x - \mu}{\sigma}$$
;
 $z_{40} = \frac{40 - 55.7}{11.5} = -1.365$;
 $z_{60} = \frac{60 - 55.7}{11.5} = 0.374$;
 $P(illegal) = 1 - P(legal) = 1 - P(40 < x < 60)$;
 $P(40 < x < 60) = P(-1.365 < z < 0.374) = P(z < 0.374) - P(z < -1.365)$;

The cumulative probability values can be either obtained from the standard normal table or calculated using the technology (NORM.S.DIST() function in MS Excel).

$$P(z < 0.374) = 0.6458;$$

 $P(z < -1.365) = 0.0861;$
 $P(40 < x < 60) = P(-1.365 < z < 0.374) = 0.6458 - 0.0861 = 0.5597;$
 $P(illegal) = 1 - 0.5597 = 0.4403.$

b. If 10% of turtles have shells longer than the given value, then 90% of turtles have shells that are shorter. The *z*-score associated with the given cumulative probability value can be either obtained from the standard

normal table or calculated using the technology (NORM.S.INV() function in MS Excel).

$$z(P=0.9)=1.282$$
;

Converting the *z*-score to the absolute value:

$$x = \mu + z\sigma$$
;
 $x = 55.7 + 1.282 \times 11.5 = 70.44$.

Answer: a. 0.4403; **b.** 70.44 cm.