Answer on Question #63075 – Math – Statistics and Probability

Question

Let X have a uniform distribution on the interval [A,B]. Compute V(X).

Solution

Since X is uniformly distributed on [A,B],

$$f(x) = \frac{1}{B-A}, \text{ if } A \le x \le B,$$

and

$$f(x) = 0, \text{ if } x < A \text{ or } x > B.$$

The expectation is

$$\mu = \int_A^B \frac{xdx}{B-A} = \frac{1}{B-A}\left(\frac{x^2}{2}\right)_A^B = \frac{1}{2}\frac{1}{B-A}(B^2 - A^2) = \frac{A+B}{2}.$$

The variance is

$$V(X) = \int_{-\infty}^{+\infty} (x-\mu)^2 f(x)dx = \int_A^B \frac{(x-\mu)^2 dx}{B-A} =$$

$$= \frac{1}{B-A}\int_A^B (x-\mu)^2 d(x-\mu) = \frac{1}{B-A} \cdot \frac{(x-\mu)^3}{3}\Bigg|_A^B$$

$$= \frac{1}{3(B-A)}[(B-\mu)^3 - (A-\mu)^3]$$

$$= \frac{1}{3}\frac{1}{B-A}\left[\left(B - \frac{A+B}{2}\right)^3 - \left(A - \frac{A+B}{2}\right)^3\right] = \frac{(B-A)^2}{12}.$$

Answer: $\frac{(B-A)^2}{12}$.