Answer on Question #63074 – Math – Statistics and Probability

Question

A table has drawers. Drawer 1 contains two red and five black biros, drawer 2 contains four red and three black biros and drawer 3 contains one red and six black biros. A drawer is chosen at random and a biro is chosen from the drawer. Find the probability that the biro is chosen from drawer 1 if the chosen biro is black.

Solution

Drawer 1	Drawer 2	Drawer 3
2 red, 5 black	4 red, 3 black	1 red, 6 black

Let A_i = 'a randomly chosen biro is from the ith drawer', where $i = 1, 2, 3$.
Let B =' a randomly chosen biro is black'.
If a drawer is chosen at random, then we have
$$P(A_1) = P(A_2) = P(A_3) = 1/3$$
The probability that the biro is black given the biro was chosen from ith drawer $(i = 1,2,3)$ will be
$$P(B|A_1) = \frac{5}{7}, \; P(B|A_2) = \frac{3}{7}, \; P(B|A_1) = \frac{6}{7}.$$
The law of total probability gives

$$P(B) = \sum_{i=1}^{3} P(A_i) \cdot P(B|A_i) =$$
$$= P(A_1) \cdot P(B|A_1) + P(A_2) \cdot P(B|A_2) + P(A_3) \cdot P(B|A_3) =$$
$$= \frac{1}{3}\left(\frac{5}{7} + \frac{3}{7} + \frac{6}{7}\right) = \frac{2}{3}$$

We know that the event B has occurred, and we want to calculate the conditional probability of the event A_1.
By Bayes' theorem, we have

$$P(A_1|B) = \frac{P(A_1) \cdot P(B|A_1)}{P(B)} = \frac{\frac{1}{3} \times \frac{5}{7}}{\frac{2}{3}} = \frac{5}{14}$$

Answer: $\frac{5}{14}$.