

Answer on Question #63049 – Math – Differential Equations

Question

In the first order differential equation

$dy/dx=f(x, y)$ the function $f(x, y)$ is a function of the ratio y/x :

$dy/dx=g(y/x)$

Show that the substitution of $U=y/x$ leads to separable equation in U and x .

Solution

$$\frac{dy}{dx} = g\left(\frac{y}{x}\right)$$

Let $\frac{y}{x} = u$, then $y = ux$ and $\frac{dy}{dx} = \frac{d(ux)}{dx} = x \frac{du}{dx} + u \frac{dx}{dx} = x \frac{du}{dx} + u$

Replacing $\frac{dy}{dx}$ with $x \frac{du}{dx} + u$ and $\frac{y}{x} = u$, we get:

$$x \frac{du}{dx} + u = g(u)$$

$$x \frac{du}{dx} = g(u) - u$$

$$\frac{du}{g(u)-u} = \frac{dx}{x}$$

Derived equation is the separable equation in u and x .

Answer: $\frac{du}{g(u)-u} = \frac{dx}{x}$.