# Answer on Question #63038 – Math – Abstract Algebra

## Question

How 2×2 matrix with components A B C D. Transform under SO(2).

## Solution

If $X = \begin{pmatrix} A & B \\ C & D \end{pmatrix}, Y = \begin{pmatrix} E & F \\ G & H \end{pmatrix}$, where $A, B, C, D, E, F, G, H, \lambda$ are real numbers, then

$$X + Y = \begin{pmatrix} A & B \\ C & D \end{pmatrix} + \begin{pmatrix} E & F \\ G & H \end{pmatrix} = \begin{pmatrix} A+E & B+F \\ C+G & D+H \end{pmatrix},$$

$$X - Y = \begin{pmatrix} A & B \\ C & D \end{pmatrix} - \begin{pmatrix} E & F \\ G & H \end{pmatrix} = \begin{pmatrix} A-E & B-F \\ C-G & D-H \end{pmatrix},$$

$$\lambda X = \lambda \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} \lambda A & \lambda B \\ \lambda C & \lambda D \end{pmatrix},$$

$$X^{-1} = \left( \begin{pmatrix} A & B \\ C & D \end{pmatrix} \right)^{-1} = \frac{1}{AD-BC} \begin{pmatrix} D & -B \\ -C & A \end{pmatrix},$$

$$XY = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} E & F \\ G & H \end{pmatrix} = \begin{pmatrix} AE+BG & AF+BH \\ CE+DG & CF+DH \end{pmatrix}.$$

Group SO(n) consists of 2×2 matrices satisfying conditions

$$Q^T Q = Q Q^T = I, \; det(Q) = 1,$$

where elements of $Q$ are real, $Q^T$ is the transpose of $Q$ and $I$ is the identity matrix, $det(Q)$ is the determinant of the matrix $Q$.

The group SO(2) consists of matrices of the form

$$\begin{pmatrix} \cos t & -\sin t \\ \sin t & \cos t \end{pmatrix},$$

where $t$ takes on real values.

If $X = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$ and $Q = \begin{pmatrix} \cos t & -\sin t \\ \sin t & \cos t \end{pmatrix}$, then

$$XQ = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} \cos t & -\sin t \\ \sin t & \cos t \end{pmatrix} = \begin{pmatrix} A\cos t + B\mathrm{sin}t & -A\mathrm{sin}t + B\mathrm{cos}t \\ C\mathrm{cos}t + D\mathrm{sin}t & -C\mathrm{sin}t + D\mathrm{cos}t \end{pmatrix},$$

$$QX = \begin{pmatrix} \mathrm{cos}t & -\mathrm{sin}t \\ \mathrm{sin}t & \mathrm{cos}t \end{pmatrix} \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} A\mathrm{cos}t - C\mathrm{sin}t & B\mathrm{cos}t - D\mathrm{sin}t \\ A\mathrm{sin}t + C\mathrm{cos}t & B\mathrm{sin}t + D\mathrm{cos}t \end{pmatrix}.$$

Other operations are performed using the general rules of matrix operations.