

Answer on Question #62920 – Math – Analytic Geometry

Question

1. Find a unit vector parallel to the resultant vector

$$\vec{A}_1 = 2\vec{i} + 4\vec{j} - 5\vec{k}, \vec{A}_2 = \vec{i} + 2\vec{j} + 3\vec{k}.$$

Solution

The resultant vector is

$$\vec{A} = \vec{A}_1 + \vec{A}_2 = (2\vec{i} + 4\vec{j} - 5\vec{k}) + (\vec{i} + 2\vec{j} + 3\vec{k}) = 3\vec{i} + 6\vec{j} - 2\vec{k}.$$

Find the magnitude of the vector \vec{A} :

$$|\vec{A}| = \sqrt{3^2 + 6^2 + (-2)^2} = \sqrt{49} = 7.$$

A unit vector parallel to the resultant vector:

$$\vec{e}_A = \pm \frac{\vec{A}}{|\vec{A}|} = \pm \frac{3\vec{i} + 6\vec{j} - 2\vec{k}}{7}.$$

$$\vec{e}_A = \left(\frac{3}{7}, \frac{6}{7}, -\frac{2}{7}\right) \text{ or } \vec{e}_A = \left(-\frac{3}{7}, -\frac{6}{7}, \frac{2}{7}\right).$$

Answer: $\vec{e}_A = \left(\frac{3}{7}, \frac{6}{7}, -\frac{2}{7}\right)$ or $\vec{e}_A = \left(-\frac{3}{7}, -\frac{6}{7}, \frac{2}{7}\right)$.

Question

2. Given the scalar defined by

$$\varphi(x, y, z) = 3x^2z - xy^2 + 5,$$

find φ at the point $(-1, -2, -3)$.

Solution

$$\varphi(-1, -2, -3) = 3(-1)^2(-3) - (-1)(-2)^2 + 5 = -9 + 4 + 5 = 0.$$

Answer: $\varphi(-1, -2, -3) = 0$.