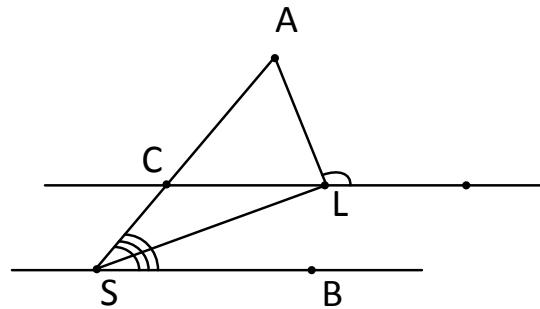## Question

From a lighthouse L an aircraft carrier A is 15km away on a bearing of 112 degrees and a submarine S is 26 km away on a bearing of 200 degrees.
Find
a) the distance between A and S
b) the bearing of A from S

Solution

a) Compute $\angle ALS = 200° - 112° = 88°$.

Using the law of cosines one finds side AS:

$c^2 = a^2 + b^2 - 2ab \cdot \cos(88°)$;

$c^2 = 15^2 + 26^2 - 2 \cdot 15 \cdot 26 \cdot \cos(88°)$;

$c^2 = 225 + 676 - 780 \cdot 0.0348995$;

$c^2 = 873.778$;

$c = 29.5597$;

So the distance between A and S is 29.5597 km.

b) Now find the angle $\angle LAS$ by the law of sines:

$$\frac{LS}{\sin \angle LAS} = \frac{AS}{\sin \angle ALS} ;$$

$$\frac{\sin(\angle LAS)}{26} = \frac{\sin(88°)}{29.5597} ;$$

$$\sin(\angle LAS) = 26 \cdot \frac{\sin(88°)}{29.5597} ;$$

$$\angle LAS = \operatorname{asin}\left(26 \cdot \frac{\sin(88°)}{29.5597}\right),$$

where asin is the inverse of the sine function;

$\angle LAS = \operatorname{asin}(0.8790400952)$;

∠LAS = 61.53°.

The bearing of A from S is angle ∠ASB:

∠ASB = ∠ASL + ∠LSB;

Now find the angle ∠ASL:

∠ASL = 180° − ∠ALS − ∠LAS = 180° − 88° − 61.53° = 30.47°.

As lines CL and SB are parallel, we know that alternate interior angles are equal, so

∠LSB = ∠SLC = 200° − 180° = 20°.

Then

∠ASB = ∠ASL + ∠LSB = 30.47° + 20° = 50.47°.

Answer: the distance between A and S is 29.5597 km, the bearing of A from S is 50.47°.