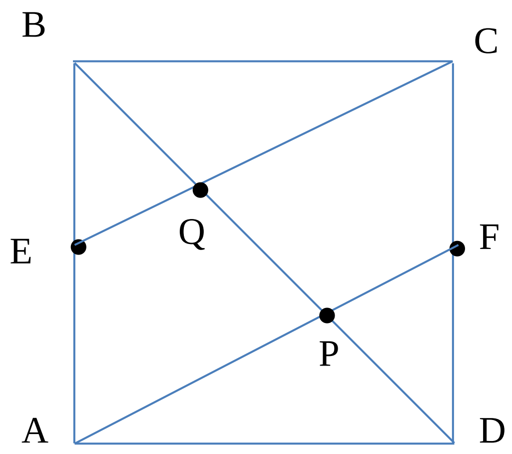
Answer on Question #62800 – Math – Geometry

Question

Pada persegi ABCD, E dan F adalah titik-titik tengah AB dan CD. Garis AF dan CE memotong diagonal BD masing-masing di P dan Q. Buktikan 3PQ=BD.

In the square ABCD, E and F are midpoints of AB and CD respectively. Lines AF and CE meet diagonal BD in points P and Q respectively. Prove that 3PQ=BD.

Solution



If *ABCD* is a square, then *BC*=*AD*, $\angle EBC = \angle FDA = 90^{\circ}$. If E and F are midpoints of AB and CD respectively, then $EB = FD$. Then $\Delta EBC \cong \Delta FDA$ by Leg-Leg (LL) Theorem.

If *ABCD* is a square, then *AB*=*DC* and $AB \parallel DC$. If *E* and *F* are midpoints of *AB*, *CD* respectively and *AB*=*DC*, then $AE = CF$. If *ABCD* is a square, then $AE \parallel CF$.

Thus, *AECF* is a parallelogram and $EC \parallel AF$. In particular, $EQ \parallel AP$.

Consider triangle ΔABP.

By the Intercept Theorem,

$$\frac{BE}{AE} = \frac{BQ}{PQ}$$

Since E is midpoint of AB, then $BE = AE$, and $\frac{BE}{AE} = 1$. So

$$\frac{BQ}{PQ} = 1,$$

that is,

$$BQ = PQ.$$

Consider ΔQCD. By the Intercept Theorem,

$$\frac{DF}{CF} = \frac{DP}{PQ}$$

Since F is midpoint of CD, then $CF = DF$ and $\frac{DF}{CF} = 1$. So

$$\frac{DP}{PQ} = 1$$

and $DP = PQ$.

Find BD:

$$BD = BQ + PQ + DP.$$

But $BQ = PQ$ and $DP = PQ$.

We finally get

$$BD = PQ + PQ + PQ = 3PQ.$$

Thus, we proved that $3PQ = BD$.