Answer on Question #62769 – Math – Trigonometry

Question

Prove

$$\frac{\sin(A+B)+\cos(A-B)}{\sin(A-B)+\cos(A+B)} = \tan(\frac{180}{4} + B) \qquad (1)$$

Solution

If we manage to transform the left side of the identity (1) into the right side, then we shall prove the identity (1). At first we simplify the numerator and denominator. Then we make a substitution.

1) $\sin(A+B)+\cos(A-B) = \sin(A+B)+\sin\left(\frac{\pi}{2}-A+B\right) = 2\ \sin\left(\frac{A+B+\frac{\pi}{2}-A+B}{2}\right)\cos\left(\frac{A+B-\frac{\pi}{2}+A-B}{2}\right)=$
$=2\sin\left(\frac{2B+\frac{\pi}{2}}{2}\right)\cos\left(\frac{2A-\frac{\pi}{2}}{2}\right) = 2\sin\left(B+\frac{\pi}{4}\right)\cos\left(A-\frac{\pi}{4}\right)$

2) $\sin(A-B)+\cos(A+B) = \sin(A-B)+\sin\left(\frac{\pi}{2}-A-B\right) = 2\ \sin\left(\frac{A-B+\frac{\pi}{2}-A-B}{2}\right)\cos\left(\frac{A-B-\frac{\pi}{2}+A+B}{2}\right) =$
$=2\sin\left(\frac{\frac{\pi}{2}-2B}{2}\right)\cos\left(\frac{2A-\frac{\pi}{2}}{2}\right) = 2\sin\left(\frac{\pi}{4}-B\right)\cos\left(A-\frac{\pi}{4}\right)$

3) $\frac{\sin(A+B)+\cos(A-B)}{\sin(A-B)+\cos(A+B)} = \frac{2\sin\left(B+\frac{\pi}{4}\right)\cos\left(A-\frac{\pi}{4}\right)}{2\sin\left(\frac{\pi}{4}-B\right)\cos\left(A-\frac{\pi}{4}\right)} = \frac{\sin\left(B+\frac{\pi}{4}\right)}{\sin\left(\frac{\pi}{4}-B\right)} = \frac{\sin\left(B+\frac{\pi}{4}\right)}{\cos\left(\frac{\pi}{2}-\left(\frac{\pi}{4}-B\right)\right)} = \frac{\sin\left(B+\frac{\pi}{4}\right)}{\cos\left(\frac{\pi}{2}-\frac{\pi}{4}+B\right)} = \frac{\sin\left(B+\frac{\pi}{4}\right)}{\cos\left(\frac{\pi}{4}+B\right)} =$
$= \tan\left(B+\frac{\pi}{4}\right)$.

4) We know that $\pi = 180^{\circ}$, so $\tan\left(B+\frac{\pi}{4}\right) = \tan\left(B+\frac{180^{\circ}}{4}\right)$.
Thus, the identity (1) has been proved.