

Answer on Question #62719 – Math – Trigonometry

Question

Solve for x :

$$3\tan^3 x - \tan x = 0$$

Solution

$$3\tan^3 x - \tan x = 0$$

$$\tan x \cdot (3\tan^2 x - 1) = 0$$

The following cases are possible:

1) $\tan(x) = 0$

$$x = \pi k, k \in \mathbb{Z}$$

2) $3\tan^2(x) - 1 = 0$

$$3\tan^2(x) = 1$$

$$\tan^2(x) = \frac{1}{3} \Leftrightarrow \tan(x) = \frac{1}{\sqrt{3}} \text{ or } \tan(x) = -\frac{1}{\sqrt{3}}$$

If $\tan(x) = \frac{1}{\sqrt{3}}$, then

$$x = \arctan\left(\frac{1}{\sqrt{3}}\right) + l\pi, l \in \mathbb{Z}$$

$$x = \frac{\pi}{6} + l\pi, l \in \mathbb{Z}.$$

If $\tan(x) = -\frac{1}{\sqrt{3}}$, then

$$x = \arctan\left(-\frac{1}{\sqrt{3}}\right) + m\pi, m \in \mathbb{Z}$$

$$x = -\frac{\pi}{6} + m\pi, m \in \mathbb{Z}.$$

Thus solutions of equation $3\tan^3 x - \tan x = 0$ are $x = \pi k, x = \frac{\pi}{6} + l\pi, x = -\frac{\pi}{6} + m\pi, k \in \mathbb{Z}, l \in \mathbb{Z}, m \in \mathbb{Z}$.

Answer: $x = \pi k, x = \frac{\pi}{6} + l\pi, x = -\frac{\pi}{6} + m\pi, k \in \mathbb{Z}, l \in \mathbb{Z}, m \in \mathbb{Z}$.