Answer on Question #62619 – Math – Statistics and Probability

Question

In a card game with my friend, I pay a certain amount of money each time I lose.
I win $4 if I draw a jack or a queen and I win $5 if I draw a king or ace for an ordinary pack of 52 playing cards.
If I draw other cards I lose.
What should I pay so that we come out even? (That is if the game is fair).

Solution

The probability mass function of amount of money is as follows

X	J, Q ($4)	K, A ($5)	Lose (-$x)
P(X)	8/52=2/13	8/52=2/13	(52-16)/52=9/13

The expectation (the expected value) of a random variable X is equal to

$$E(X) = \sum x_i P(x_i) = 4 \times \frac{2}{13} + 5 \times \frac{2}{13} - x \times \frac{9}{13} = \frac{18 - 9x}{13}.$$

Now the expected value should be $0 for the game to be fair.

$$\frac{18 - 9x}{13} = 0 => x = 2.$$

So I should pay $2 for it to be a fair game.

Answer: $2.