Answer on Question #62580 - Math - Statistics and Probability

Question

If we draw 7 cards from a standard 52 card deck, what is the probability that all 7 cards are hearts?

Solution

a) How many 7 card hands are possible?

Since the order of the cards in the hand does not matter the number of hands is

$$N = {52 \choose 7} = \frac{52!}{7! (52 - 7)!} = \frac{46 \cdot 47 \cdot 48 \cdot 49 \cdot 50 \cdot 51 \cdot 52}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7} = 133784560.$$

b) How many 7 hearts in the hand are possible?

There are 13 hearts in the deck, so the number of 7 cards hands with all hearts is

$$n_h = {13 \choose 7} = \frac{13!}{7!(13-7)!} = \frac{13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6} = 10296$$

c) What is the probability that all 7 cards are hearts?

If we draw 7 cards from a standard 52 card deck, the probability of getting all hearts is

P(all hearts) =
$$\frac{n_h}{N} = \frac{\binom{13}{7}}{\binom{52}{5}} = \frac{10296}{133784560} \approx 0.00007696.$$

Answer: $\frac{\binom{13}{7}}{\binom{52}{5}} \approx 0.00007696.$