Answer on Question #62579 – Math – Analytic Geometry

Question

Find the principal axis, vertex, focus, directrix, endpoints of the focal width and length of the focal width, and sketch the graph of the following

1. $-(x+5)^2 = y$.

Solution

1. Standard form of equation for a vertical parabola

$$(x-h)^2 = 4p(y-k)$$

(h,k) being the (x,y) coordinates of the vertex; principal axis: $x-h=0$; focus: $F(h,p+k)$; directrix: $y=-p+k$ (directrix is horizontal); length of the focal width: $d = 4|p|$; endpoints of the focal width: $B(h-2p,p+k)$, $C(h+2p,p+k)$.

For given equation $-(x+5)^2 = y$, or $(x+5)^2 = -y$ we have: $h=-5$, $k=0$, $p=-\frac{1}{4}$.

If $p=-\frac{1}{4}$, then parabola opens down. So

principal axis: $x=-5$;

vertex: $A(-5,0)$;

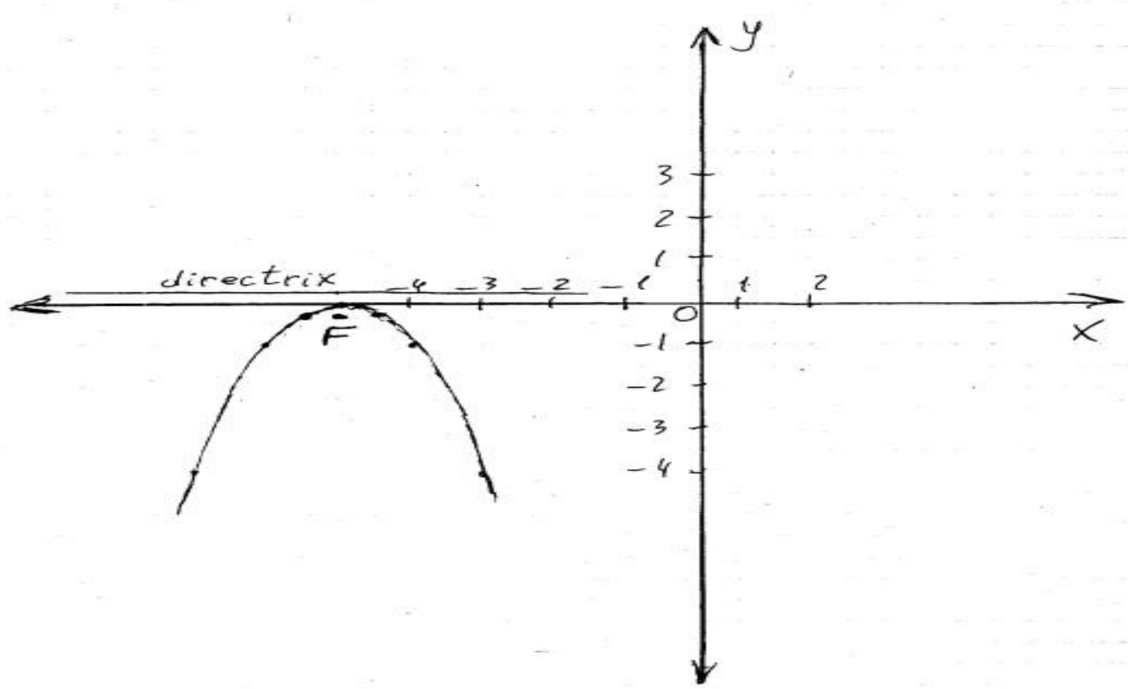
focus: $F\left(-5,-\frac{1}{4}\right)$;

directrix: $y=\frac{1}{4}$;

endpoints of the focal width: $B\left(-5\frac{1}{2},-\frac{1}{4}\right)$, $C\left(-4\frac{1}{2},-\frac{1}{4}\right)$;

length of the focal width: $d=1$.

Sketch the graph:



Answer:

principal axis: $x=-5$;

vertex: $A(-5,0)$;

focus: $\mathrm{F}\left(-5,-\frac{1}{4}\right)$;

directrix: $\mathrm{y}=\frac{1}{4}$;

endpoints of the focal width: $\mathrm{B}\left(-5\frac{1}{2},-\frac{1}{4}\right),\ \mathrm{C}\left(-4\frac{1}{2},-\frac{1}{4}\right)$;

length of the focal width: $\mathrm{d}=1$.

Question

Find the principal axis, vertex, focus, directrix, endpoints of the focal width and length of the focal width, and sketch the graph of the following

2. $(y-8)^2=24x+1$.

Solution

2. Standard form of equation for a horizontal parabola

$$(y-k)^2=4p(x-h)$$

where (h,k) is the coordinates of the vertex and p is the distance from the vertex to the focus. Principal axis: $y-k=0$; focus: $F(p+h,k)$; directrix: $x=-p+h$ (directrix is vertical) ; length of the focal width: $d=4|p|$;

endpoints of the focal width: $B(p+h,k+2p)$, $C(p+h,k-2p)$.

For given equation $(y-8)^2=24x+1$, or $(y-8)^2=24\left(x+\frac{1}{24}\right)$ we have $h=-\frac{1}{24}$, $k=8,\ p=6$. If $p=6>0$, then parabola opens right. So

principal axis: $y=8$;

vertex: $A\left(-\frac{1}{24},8\right)$;

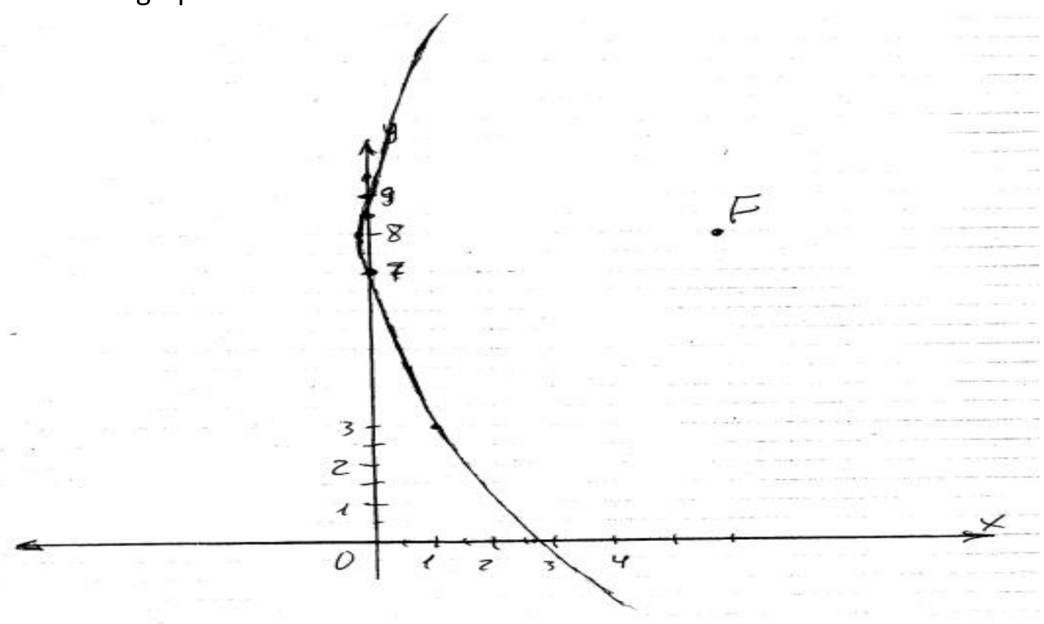
focus: $F\left(5\frac{23}{24},8\right)$;

directrix: $x=-6\frac{1}{24}$;

endpoints of the focal width: $B\left(5\frac{23}{24},20\right),\ C\left(5\frac{23}{24},-4\right)$;

length of the focal width: $d=24$.

Sketch the graph:



Answer:
principal axis: $y = 8$;
vertex: $A\left(-\frac{1}{24}, 8\right)$;
focus: $F\left(5\frac{23}{24}, 8\right)$;
directrix: $x = -6\frac{1}{24}$;
endpoints of the focal width: $B\left(5\frac{23}{24}, 20\right)$, $C\left(5\frac{23}{24}, -4\right)$;
length of the focal width: $d = 24$.

Question

Find the principal axis, vertex, focus, directrix, endpoints of the focal width and length of the focal width, and sketch the graph of the following

3. $4x - y^2 = 0$.

Solution

3. For a horizontal parabola $4x - y^2 = 0$, or $y^2 = 4x$ we have: $h = 0$, $k = 0$, $p = 1$.

Then:

principal axis: $y = 0$;

vertex: $A(0,0)$;

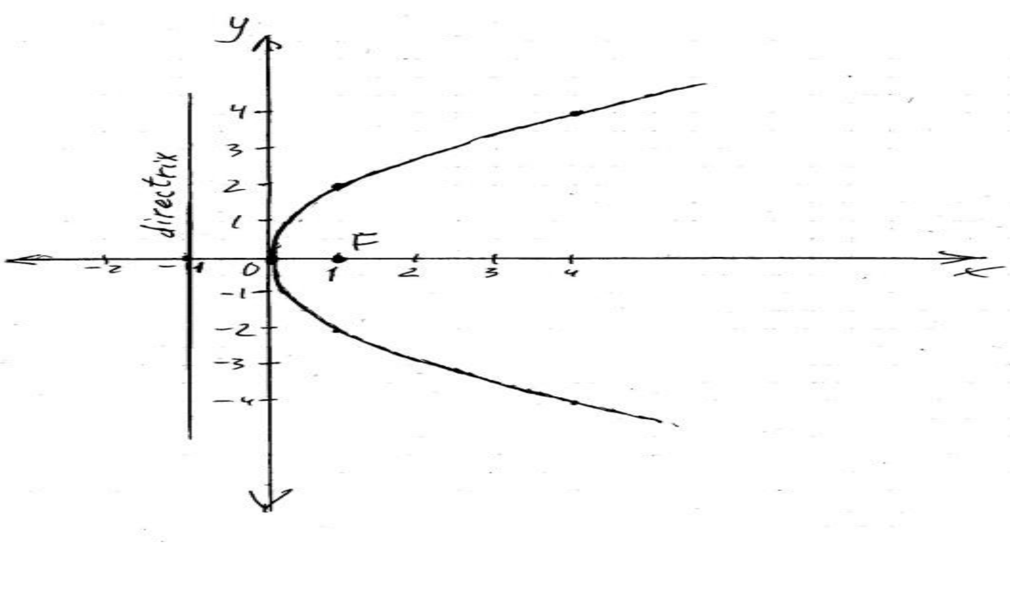
focus: $F(1,0)$;

directrix: $x = -1$ (directrix is vertical);

endpoints of the focal width: $B(1,2)$, $C(1,-2)$;

length of the focal width: $d = 4$.

Sketch the graph:



Answer:

principal axis: $y = 0$;

vertex: $A(0,0)$;

focus: $F(1,0)$;

directrix: $x = -1$;

endpoints of the focal width: $B(1,2)$, $C(1,-2)$;

length of the focal width: $d = 4$.