

Answer on Question #62483 – Math – Calculus

Question

Which of the following could be an example of a function with a domain (- infinity, infinity) and a range (- infinity, 4).

Solution

The question has more than one solution.

The quadratic function with the vertex $(x_0, 4)$ and $a < 0$ is an example of a function:

$$f(x) = ax^2 + bx + c, a < 0;$$

$$f\left(-\frac{b}{2a}\right) = a\left(-\frac{b}{2a}\right)^2 + b\left(-\frac{b}{2a}\right) + c = -\frac{b^2}{4a} + c = 4,$$

hence

$$c = 4 + \frac{b^2}{4a}.$$

Examples:

If $b = 2, a = -1$, then

$$c = 4 + \frac{b^2}{4a} = 4 + \frac{2^2}{4 \cdot (-1)} = 4 - \frac{4}{4} = 4 - 1 = 3$$

and

$$f(x) = -x^2 + 2x + 3.$$

If $b = 0, a = 1$, then

$$c = 4 + \frac{b^2}{4a} = 4 + \frac{0^2}{4 \cdot 1} = 4$$

and

$$f(x) = -x^2 + 4.$$

Answer: $f(x) = -x^2 + 2x + 3; f(x) = -x^2 + 4.$