Answer on Question #62421 – Math – Discrete Mathematics

Question

1. *If a quiz contains five multiple-choice questions, each of which has 4 choices. In how many ways can this be answered?*

Solution

It is an arrangement with repetition. There are 4 ways to choose an alternative in each question. So the quiz can be answered in

$$4^5 = 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 = 512 \text{ ways.}$$

Answer: 512.

Question

2. *How many different ways can a visiting nurse see six patients if she is to visit at most 3 of them in one day?*

Solution

It is computed by means of the rule of sum (addition principle of combinatorics) and combinations without repetitions:

$$n = \binom{6}{0} + \binom{6}{1} + \binom{6}{2} + \binom{6}{3} = \frac{6!}{6!\,0!} + \frac{6!}{1!\,5!} + \frac{6!}{2!\,4!} + \frac{6!}{3!\,3!} =$$

$$= 1 + 6 + \frac{6 \cdot 5}{2} + \frac{6 \cdot 5 \cdot 4}{3 \cdot 2} =$$

$$= 1 + 6 + 15 + 20 = 42,$$

where

$\binom{n}{k} = \frac{n!}{k!(n-k)!}, \quad n! = 1 \cdot 2 \cdot 3 \cdot \ldots \cdot n.$

It should be mentioned that a visiting nurse can see 0 or 1 or 2 or 3 patients.

Answer: 42.

Question

3. *How many different signals using 9 flags can be made if 4 are red, 3 are blue and 2 are white?*

Solution

Such questions are solved by permutations with repetition of 9 objects, where there are 4 indistinguishable red flags, 3 indistinguishable blue flags, 2 indistinguishable white flags:

$$\frac{9!}{4!\,3!\,2!} = \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5}{3 \cdot 2 \cdot 2} = 1260$$

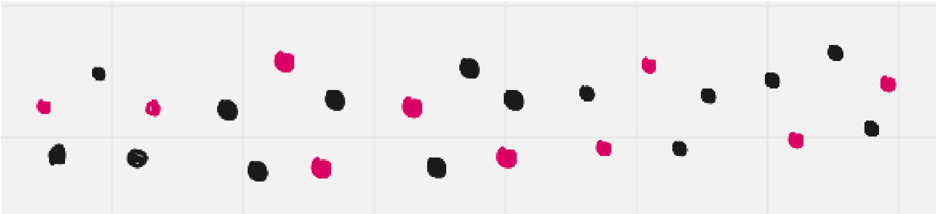
Answer: 1260.

Question

4. *In how many ways can 5 individuals be seated in a round table with 5 chairs if 2 of them refuse to sit next to each other?*

Solution

Method 1



It is a circular permutation with restriction. In this situation Black individuals can be seated in a round table with 3 chairs in 3! ways and Red individuals be seated in 2 ways.

$$3! \cdot 2 = 3 \cdot 2 \cdot 2 = 12$$

Method 2

The round table does not have a beginning nor end. Combinations 1 2 3 4 5 6 and 6 1 2 3 4 5 are completely the same. To avoid double counting 1 is fixed as the starting point. Then 5 individuals can be seated at a round table in

$$|U| = (5-1)! = 4! \text{ ways.}$$

We shall use the formula

$$|U| = |A| + |\bar{A}|,$$

where $|U|$ is the total number of combinations, $|A|$ is the number of 'wrong' combinations, $|\bar{A}|$ is the number of 'correct' combinations.

If the mentioned two individuals can sit next to each other, then we could think of them two as one person, so there will be 4 people to sit down and the mentioned two individuals can sit in two different ways. So we shall multiply the number of combinations by 2. The number of combinations where the two individuals sit next to each other is

$$|A| = (4-1)! \cdot 2 = 3! \cdot 2\ .$$

The total number of combinations to seat 5 individuals in a round table with 5 chairs if 2 of them refuse to sit next to each other will be

$$|\bar{A}| = |U| - |A| = 4! - 3!\,2 = 4 \cdot 3! - 2 \cdot 3! = (4-2) \cdot 3! = 2 \cdot 3! = \\ = 2 \cdot 3 \cdot 2 = 12$$

Answer: 12.

Question

5. *If a basketball team has 10 games to play, how many different outcomes of 6 wins and 4 losses are possible?*

Solution

Such questions are solved by permutations with repetition of 10 objects, where there are 6 indistinguishable wins and 4 indistinguishable losses:

$$\frac{10!}{6!\,4!} = \frac{10 \cdot 9 \cdot 8 \cdot 7}{4 \cdot 3 \cdot 2} = 210$$

Answer: 210.

Question

6. *There are 12 people at a party. If they all shake hands once, how many different handshakes are there?*

Solution

It is a combination without repetition. The total number of shaking hands is

$$\binom{n}{2} = \frac{n!}{(n-2)! \cdot 2!} = \frac{n(n-1)}{2} = \frac{12 \cdot (12-1)}{2} = \frac{12 \cdot 11}{2} = 66$$

Answer: 66.

Question

7. *In how many ways can 6 individuals be seated in a row if three insist on sitting next to each other?*

Solution

Method 1

It is computed by means of the rule of product (multiplication principle of combinatorics) and permutations without repetitions.

There are 4! ways to arrange a group of 3 individuals with 3 other persons. Among the three individuals (in a group), they can arrange themselves in 3! different ways. Therefore, 6 individuals can be seated in a row if three insist on sitting next to each other in

$4! \cdot 3! = 4 \cdot 3 \cdot 2 \cdot 3 \cdot 2 = 4 \cdot 3^2 \cdot 2^2 = 4 \cdot 9 \cdot 4 = 4^2 \cdot 9 = 16 \cdot 9 = 144$

ways.

Method 2

Let X be individuals who insists on sitting next to each other and O are others. So we have XXXOOO where the first X can be chosen in 3 ways, the second X in 2 ways and the third X in 1 way. There are $3 \cdot 2 \cdot 1 = 3!$ ways.

The first O can be chosen in 3 ways, the second O in 2 ways and the third O in 1 way. There are $3 \cdot 2 \cdot 1 = 3!$ ways.
There are

$$3! \cdot 3! = 36$$

ways in total.
Individuals X and O can be arranged in a row as follows:

1) X X X O O O
2) O X X X O O
3) O O X X X O
4) O O O X X X

There are 4 possibilities of arrangement.
Therefore, the number of ways will be multiplied by 4.
So,

$$4 \cdot 3! \cdot 3! = 4! \cdot 3! = 4 \cdot 3 \cdot 2 \cdot 3 \cdot 2 = 4 \cdot 3^2 \cdot 2^2 = 4 \cdot 9 \cdot 4 = 4^2 \cdot 9 =$$
$$= 16 \cdot 9 = 144.$$

Answer: 144.