Answer on Question #62407 – Math – Combinatorics | Number Theory

Question

Prove that $\sqrt{n}$ is irrational

Solution

We will prove by contradiction that if n is natural and it is not square of natural number, then $\sqrt{n}$ is irrational.
Suppose there exists some n which is not complete square, but $\sqrt{n}$ is rational.
Then we can write

$$\sqrt{n} = \frac{p}{q},$$

where p and q are coprime natural numbers.
Squaring both sides we have

$$n = \frac{p^2}{q^2} \implies nq^2 = p^2.$$

It follows from the last equality that p^2 is divisible by q and it is coprime to q.
So q obiously must be 1.
Then

$$\sqrt{n} = p \implies n = p^2,$$

which contradicts the condition 'n is not complete square'.
Therefore, assumption '$\sqrt{n}$ is rational' was false.
So we proved that $\sqrt{n}$ is irrational for those n which are not complete squares.