

## Answer on Question #62406 – Math – Abstract Algebra

### Question

What is sub-ring? Write its two examples.

### Solution

#### Definition

A subring  $S$  of a ring  $R$  is a subset of  $R$  which is a ring under the same operations as  $R$ .

Equivalently:

#### The criterion for a subring

A non-empty subset  $S$  of  $R$  is a subring if  $a, b \in S \Rightarrow a - b, ab \in S$ .

So  $S$  is closed under subtraction and multiplication.

#### Examples

1. The even integers  $2\mathbb{Z}$  form a subring of  $\mathbb{Z}$ .

More generally, if  $n$  is any integer the set of all multiples of  $n$  is a subring  $n\mathbb{Z}$  of  $\mathbb{Z}$ .

The odd integers do not form a subring of  $\mathbb{Z}$ .

2. The set  $\{a + bi \in \mathbb{C} \mid a, b \in \mathbb{Z}\}$  forms a subring of  $\mathbb{C}$ .

This is called the ring of Gaussian integers (sometimes written  $\mathbb{Z}[i]$ ) and it is important in Number Theory.