Answer on Question #62348 – Math – Analytic Geometry

Question

Find the equation to the straight line passing through the point of intersection of the lines 5x–6y –1 = 0 and 3x + 2y +5 =0 and perpendicular to the line 3x–5y+11 = 0.

Solution

Let's find the point of intersection of the lines $5x-6y-1=0$ and $3x+2y+5=0$:

$$\begin{cases} 5x-6y-1=0, \\ 3x+2y+5=0. \end{cases} \quad (1)$$

Multiply the second equation by 3:

$$\begin{cases} 5x-6y-1=0, \\ 9x+6y+15=0. \end{cases}$$

Adding two equations

$$14x+14=0.$$

Then

$$14x=-14.$$

Hence

$$x=-1.$$

Substituting $x=-1$ into the equation (1):

$$5\cdot(-1)-6y-1=0;$$
$$-6-6y=0.$$

Then

$$6y=-6.$$

Hence

$$y=-1.$$

Hence the point of intersection of the lines $5x-6y-1=0$ and $3x+2y+5=0$ is $(-1;-1)$.

The equation of the line in the form $y=kx+b$ is called the slope-intercept form, because k is the slope and b gives the *y*-intercept.

Find the slope of the line $3x-5y+11=0$.

Dividing $5y=3x+11$ by 5:

$$y=\frac{3}{5}x+\frac{11}{5}.$$

Then the slope is

$$k=\frac{3}{5}.$$

If two lines are perpendicular, their slopes *k* and k_1 are negative reciprocals.

It means

$$k\cdot k_1=-1.$$

We get

$$k_1=-\frac{1}{k}=-\frac{1}{\frac{3}{5}}=-\frac{5}{3}.$$

The general equation of the line through the point $(x_1;y_1)$ with the slope k_1 is given by

$$y - y_1 = k_1(x - x_1).$$

Let's find the equation of the line that passes through the point $(-1;-1)$ with the slope $k_1 = -\frac{5}{3}$:

$$y + 1 = -\frac{5}{3}(x + 1).$$

Multiplying by 3:

$$3y + 3 = -5(x + 1);$$
$$3y + 3 = -5x - 5.$$

The equation to the straight line is

$$5x + 3y + 8 = 0.$$

Answer: $5x + 3y + 8 = 0$.