Answer on Question #62279 – Math – Trigonometry

Question

Verify:

[tan^2x/(tan^2x-1)]+[cosec^2x/(sec^2x-cosec^2x)]=1/(sin^2x-cos^2x) **(1)**

Solution

Let's transform the left-hand side of the expression (1):

$$\frac{tan^2x}{tan^2x-1}+\frac{cosec^2x}{sec^2x-cosec^2x}=\frac{sin^2x}{cos^2x\cdot\left(\frac{sin^2x}{cos^2x}-1\right)}+\frac{\frac{1}{sin^2x}}{\frac{1}{cos^2x}-\frac{1}{sin^2x}}=$$

$$=\frac{sin^2x}{cos^2x\cdot\frac{sin^2x-cos^2x}{cos^2x}}+\frac{\frac{1}{sin^2x}}{\frac{sin^2x-cos^2x}{cos^2x\cdot sin^2x}}=\frac{sin^2x}{sin^2x-cos^2x}+\frac{1}{sin^2x}\cdot\frac{cos^2x\cdot sin^2x}{sin^2x-cos^2x}=$$

$$=\frac{sin^2x}{sin^2x-cos^2x}+\frac{cos^2x}{sin^2x-cos^2x}=\frac{sin^2x+cos^2x}{sin^2x-cos^2x}=\frac{1}{sin^2x-cos^2x}.$$

We can see that we get the right-hand side of the expression (1). It means that the identity (1) is proved.