Answer on Question #62266 – Math – Calculus

Question

Evaluate the integral $\int_0^{\pi/2} \frac{1}{2+\cos x} dx$

Solution

$$I = \int \frac{1}{2+\cos x} dx = \int \frac{1}{1 + 2\cos^2 \frac{x}{2}} dx = \int \frac{1}{\cos^2 \frac{x}{2}\left(\frac{1}{\cos^2 \frac{x}{2}} + 2\right)} dx =$$

$$= \int \frac{2}{1 + \tan^2 \frac{x}{2} + 2} d(\tan \frac{x}{2}) = \frac{2}{3} \int \frac{1}{1 + \frac{\tan^2 \frac{x}{2}}{3}} d(\tan \frac{x}{2}) =$$

$$= \frac{2}{\sqrt{3}} \int \frac{1}{1 + \left(\frac{\tan \frac{x}{2}}{\sqrt{3}}\right)^2} d\left(\frac{\tan \frac{x}{2}}{\sqrt{3}}\right) = \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{\tan \frac{x}{2}}{\sqrt{3}}\right) + C.$$

$$\int_0^{\pi/2} \frac{1}{2+\cos x} dx = \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{\tan \frac{x}{2}}{\sqrt{3}}\right) \Big|_0^{\frac{\pi}{2}} =$$

$$= \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{\tan \frac{\pi}{2 \cdot 2}}{\sqrt{3}}\right) - \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{\tan 0}{\sqrt{3}}\right) = \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) - 0 = \frac{2}{\sqrt{3}} \cdot \frac{\pi}{6} = \frac{\sqrt{3}}{9}.$$

Question

Evaluate the integral $\int_0^{\pi/2} \frac{1}{1+\cos x} dx$

Solution

$$\int_0^{\pi/2} \frac{1}{1+\cos x} dx = \int_0^{\pi/2} \frac{1}{2\cos^2 \frac{x}{2}} dx = \int_0^{\frac{\pi}{2}} \frac{2}{2\cos^2 \frac{x}{2}} d\left(\frac{x}{2}\right) = \tan \frac{x}{2} \Big|_0^{\frac{\pi}{2}} =$$

$$= \tan \frac{\pi}{4} - \tan 0 = 1 - 0 = 0.$$