Answer on Question #62265 – Math – Analytic Geometry

Question

Find the equation of the sphere which passes through the points (0,3,1), (2,5,1), (3,4,1,)
and whose center lies on the plane 0 =x+y+z.

Solution

Let x, y, z be coordinates of the center of the sphere.

$$\begin{cases} \sqrt{(x-0)^2+(y-3)^2+(z-1)^2} = \sqrt{(x-2)^2+(y-5)^2+(z-1)^2} = \sqrt{(x-3)^2+(y-4)^2+(z-1)^2} \\ x+y+z=0 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} x^2+(y-3)^2+(z-1)^2 = (x-2)^2+(y-5)^2+(z-1)^2 \\ x^2+(y-3)^2+(z-1)^2 = (x-3)^2+(y-4)^2+(z-1)^2 \\ x+y+z=0 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} x^2+y^2-6y+9+z^2-2z+1 = x^2-4x+4+y^2-10y+25+z^2-2z+1 \\ x^2+y^2-6y+9+z^2-2z+1 = x^2-6x+9+y^2-8y+16+z^2-2z+1 \\ x+y+z=0 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} -6y-2z+10 = -4x-10y-2z+30 \\ -6y-2z+10 = -6x-8y-2z+26 \\ x+y+z=0 \end{cases} \Rightarrow \begin{cases} 4x+4y=20 \\ 6x+2y=16 \\ x+y+z=0 \end{cases} \Rightarrow \begin{cases} y=5-x \\ 6x+10-2x=16 \\ x+5-x+z=0 \end{cases} \Rightarrow \begin{cases} y=5-x \\ 4x=6 \\ z=-5 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} x=1.5 \\ y=3.5 \\ z=-5 \end{cases}$$

We found the center of the sphere is $(1.5, 3.5, -5)$.

The radius of the sphere is

$$r = \sqrt{(1,5-0)^2+(3,5-3)^2+(-5-1)^2} = \sqrt{2,25+0,25+36} = \sqrt{38.5} \Rightarrow r^2 = 38.5.$$

The equation of the sphere is

$$(x-1.5)^2+(y-3.5)^2+(z+5)^2 = 38.5.$$

Answer: $(x-1.5)^2+(y-3.5)^2+(z+5)^2 = 38.5$.