Answer on Question #62240 – Math – Algebra

Question

$$7x - y + z = -24,$$
$$2x + 2y - 3z = -22,$$
$$x - 3y + 2z = 2.$$

Solution

It follows from the third equation $x - 3y + 2z = 2$ that

$$x = 3y - 2z + 2\,. \quad \textbf{(1)}$$

Plug (1) for x in the second equation $2x + 2y - 3z = -22$:

$$2(3y - 2z + 2) + 2y - 3z = -22,$$

$$6y - 4z + 4 + 2y - 3z = -22.$$

Subtract 4 from both sides:

$$6y - 4z + 2y - 3z = -26.$$

Collect the like terms and simplify:

$$6y + 2y - 4z - 3z = -26,$$

$$8y - 7z = -26.$$

Add $7z$ to both sides:

$$8y = 7z - 26.$$

Divide by 8:

$$y = \frac{7z-26}{8}\,. \quad \textbf{(2)}$$

Plug (2) for y in (1):

$$x = 3y - 2z + 2 = 3\frac{7z-26}{8} - 2z + 2 = \frac{3(7z-26)-2z\cdot 8+2\cdot 8}{8} = \frac{21z-78-16z+16}{8} = \frac{5z-62}{8},$$

i.e.,

$$x = \frac{5z-62}{8}\,. \quad \textbf{(3)}$$

Plug (3) for x and (2) for y in the first equation $7x - y + z = -24$:

$$7\frac{5z-62}{8} - \frac{7z-26}{8} + z = -24. \quad \textbf{(4)}$$

Multiply (4) by 8:

$$7(5z - 62) - (7z - 26) + 8z = -192.$$

Open brackets:

$$35z - 434 - 7z + 26 + 8z = -192.$$

Add 434 to both sides and subtract 26 from both sides:

$$35z - 7z + 8z = -192 + 434 - 26.$$

Simplify:

$$36z = 216.$$

Divide through by 36:

$$z = 6. \quad \textbf{(5)}$$

Substitute (5) for z into (3):

$$x = \frac{5z-62}{8} = \frac{5\cdot 6-62}{8} = \frac{30-62}{8} = \frac{-32}{8} = -4.$$

Substitute (5) for z into (2):

$$y = \frac{7z-26}{8} = \frac{7\cdot 6-26}{8} = \frac{42-26}{8} = \frac{16}{8} = 2.$$

Thus,

$$x = -4, y = 2, z = 6.$$

Answer: $x = -4, y = 2, z = 6.$