# Answer on Question #62217 – Math – Calculus

Show that for any vector field F

$$\nabla\cdot(\nabla\times F) = 0$$

## Solution

If F = P**i** +Q **j**+ R**k** is a vector field in $R^3$, then div curl **F** = 0.

Using the definition of divergence and curl obtain:

$$\textit{div curl } \mathbf{F} = \nabla\cdot \textit{ curl } \mathbf{F} = \nabla\cdot(\nabla\times F) = \left(\frac{\partial}{\partial x}\mathbf{i} + \frac{\partial}{\partial y}\mathbf{j} + \frac{\partial}{\partial z}\mathbf{k}\right)\cdot\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ P & Q & R\end{vmatrix} =$$

$$= \left(\frac{\partial}{\partial x}\mathbf{i} + \frac{\partial}{\partial y}\mathbf{j} + \frac{\partial}{\partial z}\mathbf{k}\right)\cdot\left[\left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}\right)\mathbf{i} - \left(\frac{\partial R}{\partial x} - \frac{\partial P}{\partial z}\right)\mathbf{j} + \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)\mathbf{k}\right] =$$

$$= \frac{\partial}{\partial x}\left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}\right) - \frac{\partial}{\partial y}\left(\frac{\partial R}{\partial x} - \frac{\partial P}{\partial z}\right) + \frac{\partial}{\partial z}\left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) = \left(\frac{\partial^2 R}{\partial x\partial y} - \frac{\partial^2 Q}{\partial x\partial z}\right) - \left(\frac{\partial^2 R}{\partial y\partial x} - \frac{\partial^2 P}{\partial y\partial z}\right) + \left(\frac{\partial^2 Q}{\partial z\partial x} - \frac{\partial^2 P}{\partial z\partial y}\right) = \left(\frac{\partial^2 R}{\partial x\partial y} - \frac{\partial^2 R}{\partial y\partial x}\right) + \left(\frac{\partial^2 Q}{\partial z\partial x} - \frac{\partial^2 Q}{\partial x\partial z}\right) + \left(\frac{\partial^2 P}{\partial y\partial z} - \frac{\partial^2 P}{\partial z\partial y}\right) = 0 + 0 + 0 =$$

$= 0$, because the distributive property was applied,

$\mathbf{i}\cdot\mathbf{i} = \mathbf{j}\cdot\mathbf{j} = \mathbf{k}\cdot\mathbf{k} = 1,\ \mathbf{i}\cdot\mathbf{j} = \mathbf{i}\cdot\mathbf{k} = \mathbf{j}\cdot\mathbf{i} = \mathbf{j}\cdot\mathbf{k} = \mathbf{k}\cdot\mathbf{i} = \mathbf{k}\cdot\mathbf{j} = 0$ and

by Clairaut's heorem (or Schwarz's theorem), terms can be reduced because of the symmetry of second derivatives.