Answer on Question #62214 – Math – Statistics and Probability

Question

The measurements of the bulk modulus of a material at different temperatures is as follows:

T (^{0}C)	20	500	1000	1200	1400	1500
K (G Pa)	203	197	191	188	186	184

Determine the regression equation for this data.

Solution

We shall derive the linear regression equation, i.e. we must calculate the coefficients a and b in the equation $K = a + bT$. Now we calculate the next values:

$$\overline{T} = \frac{\sum_{i=1}^{6} t_i}{6} = \frac{20+500+1000+1200+1400+1500}{6} = 936.6667;$$

$$\overline{K} = \frac{\sum_{i=1}^{6} k_i}{6} = \frac{203+197+191+188+186+184}{6} = 191.50;$$

$$\overline{TK} = \frac{\sum_{i=1}^{6} t_i k_i}{6} = \frac{20\cdot 203+500\cdot 197+1000\cdot 191+1200\cdot 188+1400\cdot 186+1500\cdot 184}{6} = 175926.67;$$

$$\sigma_T^2 = \frac{1}{6}\sum_{i=1}^{6} (t_i - \overline{T})^2 =$$

$$= \frac{(20-936.67)^2 + (500-936.67)^2 + (1000-936.67)^2 + (1200-936.67)^2 + (1400-936.67)^2}{6} +$$

$$+ \frac{(1500-936.67)^2}{6} = 272722.22.$$

Now we can obtain the coefficients using the next formulas:

$$b = \frac{\overline{TK} - \overline{T}\cdot\overline{K}}{\sigma_T^2} = \frac{175926.67 - 936.6667\cdot 191.50}{272722.22} = -0.01263;$$

$$a = \overline{K} - b\overline{T} = 191.50 - (-0.01263)\cdot 936.6667 = 203.33.$$

The regression equation for the given data is $K = 203.33 - 0.01 \cdot T$.

Answer: $K = 203.33 - 0.01 \cdot T$.