Answer on Question #62173 – Math – Discrete Mathematics

Question

Determine the number of positive integers x, where $x < 100$ and x is divisible by 2, 3 or 5.

Solution

Method 1

The first positive integer divisible by 2 is $2 = 2 \cdot 1$. Solving inequality $2k \leq 99$, where k is integer, we get the number of positive integers less than 100 and divisible by 2.

The $[y]$ means the integer part of y (the largest integer less than or equal to y).

Let us find the number of positive integers x_1, where x_1 is divisible by 2:

$$N(A) = \left[\frac{99}{2}\right] = [49.5] = 49.$$

Further we shall find the number of positive integers x_2, where x_2 is divisible by 3:

$$N(B) = \left[\frac{99}{3}\right] = 33.$$

In the next step we shall find the number of positive integers x_3, where x_3 is divisible by 5:

$$N(C) = \left[\frac{99}{5}\right] = [19.8] = 19.$$

Then we shall determine the number of positive integers x_4, where x_4 is divisible by $6 = 3 \cdot 2$:

$$N(A \cap B) = \left[\frac{99}{6}\right] = [16.5] = 16.$$

Then we shall determine the number of positive integers x_5, where x_5 is divisible by $10 = 2 \cdot 5$:

$$N(A \cap C) = \left[\frac{99}{10}\right] = [9.9] = 9.$$

Then we shall determine the number of positive integers x_6, where x_6 is divisible by $15 = 3 \cdot 5$:

$$N(B \cap C) = \left[\frac{99}{15}\right] = [6.6] = 6.$$

Now we shall determine the number of positive integers x_7, where x_7 is divisible by

$30 = 2 \cdot 3 \cdot 5$:

$$N(A \cap B \cap C) = \left[\frac{99}{30}\right] = [3.3] = 3.$$

Method 2

We shall use the general formula for the l-th term of the arithmetic sequence:

$a_l = a_1 + (l - 1)d,$

hence

$l = \frac{a_l - a_1}{d} + 1.$

The first positive integer divisible by 2 is $a_1 = 2$, the last positive integer less than 100 and divisible by 2 is $a_l = 98$, hence the number of positive integers x_1, where x_1 is divisible by 2:

$$N(A) = \frac{98 - 2}{2} + 1 = \frac{96}{2} + 1 = 49.$$

The first positive integer divisible by 3 is $b_1 = 2$, the last positive integer less than 100 and divisible by 3 is $b_l = 99$, hence the number of positive integers x_2, where x_2 is divisible by 3:

$$N(B) = \frac{99 - 3}{3} + 1 = \frac{96}{3} + 1 = 33.$$

The first positive integer divisible by 5 is $c_1 = 5$, the last positive integer less than 100 and divisible by 5 is $c_l = 95$, hence the number of positive integers x_3, where x_3 is divisible by 5:

$$N(C) = \frac{95 - 5}{5} + 1 = \frac{90}{5} + 1 = 19.$$

The first positive integer divisible by $6 = 2 \cdot 3$ is $d_1 = 6$, the last positive integer less than 100 and divisible by 6 is $d_l = 96$, hence the number of positive integers x_4, where x_4 is divisible by 6:

$$N(A \cap B) = \frac{96 - 6}{6} + 1 = \frac{90}{6} + 1 = 16.$$

The first positive integer divisible by $10 = 2 \cdot 5$ is $e_1 = 10$, the last positive integer less than 100 and divisible by 10 is $e_l = 90$, hence the number of positive integers x_5, where x_5 is divisible by 10:

$$N(A \cap C) = \frac{90 - 10}{10} + 1 = \frac{80}{10} + 1 = 9.$$

The first positive integer divisible by $15 = 3 \cdot 5$ is $f_1 = 15$, the last positive integer less than 100 and divisible by 15 is $f_l = 90$, hence the number of positive integers x_6, where x_6 is divisible by 15:

$$N(B \cap C) = \frac{90 - 15}{15} + 1 = \frac{75}{15} + 1 = 6.$$

The first positive integer divisible by $30 = 2 \cdot 3 \cdot 5$ is $g_1 = 30$, the last positive integer less than 100 and divisible by 30 is $g_l = 90$, hence the number of positive integers x_7, where x_7 is divisible by 30:

$$N(A \cap B \cap C) = \frac{90 - 30}{30} + 1 = \frac{60}{30} + 1 = 3.$$

Finally using the inclusion-exclusion principle we determine the required value:

$$N(A \cup B \cup C) = N(A) + N(B) + N(C) - N(A \cap B) - N(A \cap C) - N(B \cap C) + N(A \cap B \cap C)$$

$$= 49 + 33 + 19 - 16 - 9 - 6 + 3 = 73.$$

Answer: 73.