

## Answer on Question #62166 – Math – Trigonometry

### Question

$$(\sin(x+3y) + \sin(3x+y)) \div (\sin 2x + \sin 2y) = 2\cos(x+y)$$

### Solution

We shall use the following formulas:

$$\sin \alpha + \sin \beta = 2 \sin \frac{\alpha+\beta}{2} \cdot \cos \frac{\alpha-\beta}{2}, \quad (1)$$

$$\sin(2\alpha) = 2 \sin(\alpha) \cos(\alpha), \quad (2)$$

$$\cos(-\alpha) = \cos(\alpha). \quad (3)$$

Then

$$\begin{aligned} (\sin(x+3y) + \sin(3x+y)) &= |(1)| = 2 \sin \frac{x+3y+3x+y}{2} \cdot \cos \frac{x+3y-3x-y}{2} = \\ &= 2 \sin(2x+2y) \cdot \cos(-x+y) = |(3)| = 2 \sin(2x+2y) \cdot \cos(x-y) = |(2)| = \\ &= 4 \sin(x+y) \cos(x+y) \cos(x-y); \end{aligned}$$

$$\sin(2x) + \sin(2y) = |(1)| = 2 \sin \frac{2x+2y}{2} \cdot \cos \frac{2x-2y}{2} = 2 \sin(x+y) \cdot \cos(x-y).$$

Thus,

$$\frac{\sin(x+3y)+\sin(3x+y)}{\sin(2x)+\sin(2y)} = \frac{4 \sin(x+y) \cos(x+y) \cos(x-y)}{2 \sin(x+y) \cdot \cos(x-y)} = 2 \cos(x+y).$$

Q.E.D.