

## Answer on Question #62165 -- Math – Trigonometry

### Question

1. If  $\tan x = a$  and  $\tan y = b$ .

Show that

$$\sin(x + y) = \frac{(a+b)}{\sqrt{(1+a^2)(1+b^2)}}.$$

### Solution

Using

$$\sin(x + y) = \sin x \cos y + \cos x \sin y,$$

$$\tan x = \frac{\sin x}{\cos x} = a,$$

$$\tan y = \frac{\sin y}{\cos y} = b,$$

$$\sin^2 x + \cos^2 x = 1,$$

$$\sin^2 y + \cos^2 y = 1,$$

rewrite

$$\begin{aligned} \sin(x + y) &= \sin x \cos y + \cos x \sin y = \frac{(\sin x \cos y + \cos x \sin y)}{\sqrt{((\cos x)^2 + (\sin x)^2)((\cos y)^2 + (\sin y)^2)}} = \\ &= \left| \text{dividing the numerator and the denominator by } \cos x \cos y \right| = \frac{\left(\frac{\sin x}{\cos x} + \frac{\sin y}{\cos y}\right)}{\sqrt{\left(1 + \left(\frac{\sin x}{\cos x}\right)^2\right) \left(1 + \left(\frac{\sin y}{\cos y}\right)^2\right)}} = \\ &= \frac{\tan x + \tan y}{\sqrt{(1 + \tan^2 x)(1 + \tan^2 y)}} = \frac{a + b}{\sqrt{(1 + a^2)(1 + b^2)}}. \end{aligned}$$

Q.E.D.