Answer on Question #62050 – Math – Abstract Algebra

Question

Which of the following are binary operations. Justify your answer.
i) The operation · defined on Q by a·b = a(b − 1).
ii) The operation · defined on [0,π] by x·y = cosxy.
Also, for those operations which are binary operations, check whether they are associative and commutative.

Solution

A binary operation f(x,y) is an operation that applies to two quantities or expressions x and y.

A binary operation on a nonempty set A is a map f:A×A->A such that

1. f is defined for every pair of elements in A, and

2. f uniquely associates each pair of elements in A to some element of A.

i) The operation · defined on Q by a·b = a(b − 1) **is binary**

The operation is defined for all a,b∈Q. For any a,b ∈Q , a(b –1)∈Q.

If $a=a_1$ and $b=b_1$, hence $a(b-1)=a_1(b_1-1)$

There exist a and b in Q such that a(b-1)≠(a-1)b, hence the operation is **not commutative.** For example: $4(\frac{3}{5}-1) \neq \frac{3}{5}(4-1)$

There exist a, b and c in Q such that a(b(c – 1) – 1) ≠(a(b – 1))(c – 1) , hence the operation is **not associative**. For example: $\frac{3}{5}(2(3-1)-1) \neq \frac{3}{5}(2-1)(3-1)$

Answer: The operation · defined on Q by a·b = a(b − 1) is binary, not associative and not commutative.

ii) The operation · defined on [0,π] by x·y = cosxy is not binary. Because there exist a and b in [0,π] such that cos ab ∉ [0,π].
For example, $\cos \pi \cdot 1 = -1 \notin [0,\pi]$.

Answer: The operation · defined on [0,π] by x·y = cosxy is not binary.