Answer on Question #61962 – Math – Calculus

Question

7 Determine ∫x2+1(x+2)3

ln(x+2)+4x+2−52(x+3)2+c

ln(x+2)−4x+2−52(x+3)2+c

−ln(x+2)−4x+2−52(x+3)2+c

ln(x−2)+4x−2−52(x+3)2+c

Solution

Method 1

$\frac{x^2+1}{(x+2)^3} = \frac{A}{(x+2)^3} + \frac{B}{(x+2)^2} + \frac{C}{x+2}$,

where

$A = (-2)^2 + 1 = 4 + 1 = 5$,

$B = (x^2 + 1)'|_{x=-2} = (2x)|_{x=-2} = 2 \cdot (-2) = -4$,

$C = \frac{1}{2}(x^2 + 1)''|_{x=-2} = \frac{1}{2}(2x)'|_{x=-2} = \frac{1}{2} \cdot 2 = 1$.

Thus, $\frac{x^2+1}{(x+2)^3} = \frac{A}{(x+2)^3} + \frac{B}{(x+2)^2} + \frac{C}{x+2} = \frac{5}{(x+2)^3} - \frac{4}{(x+2)^2} + \frac{1}{x+2}$

Method 2

$\frac{x^2+1}{(x+2)^3} = \frac{A}{(x+2)^3} + \frac{B}{(x+2)^2} + \frac{C}{x+2} = \frac{A+B(x+2)+C(x+2)^2}{(x+2)^3}$,

hence $A + B(x + 2) + C(x + 2)^2 = x^2 + 1$

If $x = -2$, then $A = (-2)^2 + 1 = 4 + 1 = 5$, so

$5 + B(x + 2) + C(x + 2)^2 = x^2 + 1$, hence

$B(x + 2) + C(x + 2)^2 = x^2 + 1 - 5$,

$B(x + 2) + C(x + 2)^2 = x^2 - 4$.

If $x = -1$, then $B + C = (-1)^2 - 4 = 1 - 4 = -3$.

If $x = 0$, then $2B + 4C = 0^2 - 4 = 0 - 4 = -4$.

Solving the system

$$\begin{cases} B + C = -3, \\ 2B + 4C = -4, \end{cases}$$

Dividing the second equation by 2

$$\begin{cases} B + C = -3, \\ B + 2C = -2, \end{cases}$$

Subtracting the first equation from the second one

$$\begin{cases} B + C = -3, \\ B + 2C - (B + C) = -2 - (-3), \end{cases}$$

$$\begin{cases} B + C = -3, \\ C = 1, \end{cases}$$

Plugging $C = 1$ into the first equation

$$\begin{cases} B + 1 = -3, \\ C = 1, \end{cases}$$

$$\begin{cases} B = -3 - 1, \\ C = 1, \end{cases}$$

$$\begin{cases} B = -4, \\ C = 1, \end{cases}$$

Thus, $\frac{x^2+1}{(x+2)^3} = \frac{A}{(x+2)^3} + \frac{B}{(x+2)^2} + \frac{C}{x+2} = \frac{5}{(x+2)^3} - \frac{4}{(x+2)^2} + \frac{1}{x+2}$

<u>Method 3</u>

$$\frac{x^2+1}{(x+2)^3} = \frac{A}{(x+2)^3} + \frac{B}{(x+2)^2} + \frac{C}{x+2}$$

$$x^2 + 1 = x^2 + 4x - 4x + 4 - 4 - 8 + 8 + 1 = x^2 + 4x + 4 - 4x - 8 - 4 + 8 + 1 =$$
$$= (x^2 + 4x + 4) - 4(x + 2) + (8 - 4 + 1) =$$

$$= (x + 2)^2 - 4(x + 2) + 5$$

Using synthetic division

x_0	x^2	x	x^0
-2	1	0	1
-2	1	$0 + (-2) \cdot 1 = -2$	$= 1 + (-2) \cdot (-2) = \mathbf{5}$
-2	1	$= -2 + (-2) \cdot 1 = \mathbf{-4}$	
-2	**1**		

Hence

$$\frac{x^2+1}{(x+2)^3} = \frac{(x+2)^2 - 4(x+2) + 5}{(x+2)^3} = \frac{(x+2)^2}{(x+2)^3} - 4\frac{x+2}{(x+2)^3} + 5\frac{1}{(x+2)^3} =$$

$= \frac{1}{x+2} - 4 \cdot \frac{1}{(x+2)^2} + \frac{5}{(x+2)^3}$.

Finally obtain

$\int \frac{x^2+1}{(x+2)^3} dx = \int \left(\frac{1}{x+2} - \frac{4}{(x+2)^2} + \frac{5}{(x+2)^3}\right) d(x+2) = \ln(x+2) + \frac{4}{x+2} - \frac{5}{2(x+2)^2} + c,$

where c is an integration constant.

Answer: $\ln(x+2)+\frac{4}{x+2}-\frac{5}{2(x+2)^2}+c.$

Question

8 Find the volume of a sphere generated by a semicircle $y=(\sqrt{r^2-x^2})$ revolving around the x-axis

−π−r32

4πr32

πr34

4πr33

Solution

$V=\int_{-r}^{r}\pi y^2dx=\int_{-r}^{r}\pi(r^2-x^2)dx=\pi\left(r^2x-\frac{x^3}{3}\right)_{-r}^{r}=\pi\left(\frac{2}{3}r^3\right)-\pi\left(-\frac{2}{3}r^3\right)=\frac{4}{3}\pi r^3.$

Answer: $\frac{4}{3}\pi r^3.$

Question

9 Evaluate ∫x2e3x

e3x3(x2−2x3+29)+c

−e3x3(x2+2x3−29)+c

e2x3(x3−x4+29)+c

ex3(x2+2x3−25)+c

Solution

Using integration by parts

$\int x^2e^{3x}\,dx=\frac{1}{3}x^2e^{3x}-\int\frac{2}{3}xe^{3x}\,dx=\frac{1}{3}x^2e^{3x}-\frac{2}{3}\int xe^{3x}dx=\frac{1}{3}x^2e^{3x}-\frac{2}{3}\left[\frac{1}{3}xe^{3x}-\int\frac{1}{3}e^{3x}dx\right]=$

$=\frac{1}{3}x^2e^{3x}-\frac{2}{3}\left[\frac{1}{3}xe^{3x}-\frac{1}{9}e^{3x}\right]+c=\frac{1}{3}x^2e^{3x}-\frac{2}{9}xe^{3x}+\frac{2}{27}e^{3x}+c=\frac{9}{27}x^2e^{3x}-\frac{6}{27}xe^{3x}+\frac{2}{27}e^{3x}+c=$
$=\frac{1}{27}(9x^2-6x+2)e^{3x}+c,$

where c is an integration constant.

Answer: $\frac{1}{27}(9x^2-6x+2)e^{3x}+\mathbf{c}$

Question

10 Given f(x) = (7x4−5x3), evaluate df(x)dx

7x4−5x3

2x3−15x2

28x2−15x2

28x3−15x2

Solution

$\frac{\mathrm{df(x)}}{\mathrm{dx}} = \frac{d}{dx}(7\mathrm{x}^4 - 5\mathrm{x}^3) = 7\frac{d}{dx}(\mathrm{x}^4) - 5\frac{d}{dx}(\mathrm{x}^3) = 7\cdot(4x^3) - 5\cdot(3x^2) = 28x^3 - 15x^2.$

Answer: $28x^3 - 15x^2.$