Answer on Question #61961 – Math – Calculus

Question

4 Find the ∫tan3xsec3xdx

tan2x+1

cot2x+1

sec2x+1

sec2x

Solution

$$I = \int \sec^3 x \tan^3 x \, dx = \int \sec^2 x \tan^2 x \tan x \sec x \, dx = \int \sec^2 x \, (\sec^2 x - 1) \tan x \sec x \, dx$$

Let $u = \sec x = \frac{1}{\cos x}$. Then

$$\frac{du}{dx} = \sec x \tan x = \frac{\sin x}{\cos^2 x},$$

$$du = \sec x \tan x \, dx.$$

$I = \int u^2(u^2 - 1)du = \int (u^4 - u^2)du = \frac{u^5}{5} - \frac{u^3}{3} + c = \frac{(\sec x)^5}{5} - \frac{(\sec x)^3}{3} + c,$

where c is an integration constant.

$$J = \int \sec(3x) \cdot \tan(3x) dx = \frac{1}{3} \int \sec(3x) \cdot \tan(3x) \, d(3x) = |v = 3x| =$$

$$= \frac{1}{3} \int \sec(v) \cdot \tan(v) dv = \frac{1}{3} \int \frac{1}{\cos(v)} \cdot \frac{\sin(v)}{\cos(v)} dv$$

$$= \frac{1}{3} \int \frac{\sin(v)}{\cos^2(v)} dv = |t = \cos(v), \qquad dt = -\sin(v) \, dv| =$$

$$= -\frac{1}{3} \int \frac{dt}{t^2} = \frac{1}{3t} + c = \frac{1}{3\cos(v)} + c = \frac{1}{3\cos(3x)} + c,$$

where c is an integration constant.

Question

5 Find ∫sec3xtanxdx

sin2x+c

cos2x+c

sec3x3+c

cos3x2+c

Solution

$$I = \int \sec^3 x \tan x dx = \int \sec^2 x \tan x \sec x dx$$

Let $u = \sec x$. Then

$$\frac{du}{dx} = \sec x \tan x$$

$$du = \sec x \tan x \, dx$$

So,

$$I = \int u^2 du = \frac{u^3}{3} + c = \frac{1}{3} \sec^3 x + c,$$

where c is an integration constant.

Answer: $\frac{1}{3} \sec^3 x + c.$

Question

6 Evaluate ∫x+1x2−3x+2dx

3ln(x+2)−2ln(x+1)+c

3ln(x−2)−2ln(x−1)+c

−3ln(x−2)−2ln(x−1)+c

3ln(x−2)+2ln(x+1)+c

Solution

Method 1

$$\frac{x+1}{x^2-3x+2} = \frac{x+1}{(x-1)(x-2)} = \frac{1+1}{1-2} \cdot \frac{1}{x-1} + \frac{2+1}{2-1} \cdot \frac{1}{x-2} = \frac{2}{-1} \cdot \frac{1}{x-1} + \frac{3}{1} \cdot \frac{1}{x-2} = \frac{3}{x-2} - \frac{2}{x-1}.$$

Method 2

$$\frac{x+1}{x^2-3x+2} = \frac{x+1}{(x-1)(x-2)} = \frac{A}{x-1} + \frac{B}{x-2} = \frac{A(x-2)+B(x-1)}{(x-1)(x-2)} = \frac{Ax-2A+Bx-B}{(x-1)(x-2)} =$$

$= \frac{(A+B)x-(2A+B)}{(x-1)(x-2)}$,

hence

$\begin{cases} A+B=1, \\ -(2A+B)=1, \end{cases}$

$\begin{cases} A=1-B, \\ -2A-B=1, \end{cases}$

$\begin{cases} A=1-B, \\ -2(1-B)-B=1, \end{cases}$

$\begin{cases} A=1-B, \\ -2+2B-B=1, \end{cases}$

$\begin{cases} A=1-B, \\ B=1+2, \end{cases}$

$\begin{cases} A=1-B, \\ B=3, \end{cases}$

$\begin{cases} A=1-3, \\ B=3, \end{cases}$

$\begin{cases} A=-2, \\ B=3. \end{cases}$

Thus,

$\frac{x+1}{x^2-3x+2} = \frac{x+1}{(x-1)(x-2)} = \frac{A}{x-1} + \frac{B}{x-2} = -\frac{2}{x-1} + \frac{3}{x-2}$.

Method 3

$$\frac{x+1}{x^2-3x+2} = \frac{x+1}{(x-1)(x-2)} = \frac{x-1+2}{(x-1)(x-2)} = \frac{x-1}{(x-1)(x-2)} + \frac{2}{(x-1)(x-2)} =$$

$= \frac{1}{x-2} + 2 \cdot \left(\frac{1}{x-2} - \frac{1}{x-1}\right) = \frac{1}{x-2} + \frac{2}{x-2} - \frac{2}{x-1} = \frac{3}{x-2} - \frac{2}{x-1}$.

Finally obtain

$$\int \frac{x+1}{x^2-3x+2}dx = \int \frac{x+1}{(x-2)(x-1)}dx = \int \left(\frac{3}{x-2} - \frac{2}{x-1}\right)dx =$$
$$= 3\int \frac{1}{x-2}d(x-2) - 2\int \frac{1}{x-1}d(x-1) =$$

$= 3\ln(x-2) - 2\ln(x-1) + c = \ln\frac{(x-2)^3}{(x-1)^2} + c$,

where c is an integration constant.

Thus, the correct answer is $3\ln(x-2) - 2\ln(x-1) + c$.

Answer: $3\ln(x-2) - 2\ln(x-1) + c$.