Answer on Question #61835 – Math – Algebra

Question #1

Solve the following equation, giving answer as integer, surd or fraction in its simplest form

$14x - 3 = 2x + 21$

Solution

$14x - 3 = 2x + 21$

Add 3 to both sides of the equation:
$14x - 3 + 3 = 2x + 21 + 3$

Simplify:

$14x = 2x + 24$

Subtract $2x$ *from both sides of the equation:*

$14x - 2x = 2x - 2x + 24$

Simplify:

$12x = 24$

Divide by 12 to isolate x:

$x = 2$

Answer: $x = 2$

Question #2

Solve the following equation, giving answer as integer, surd or fraction in its simplest form

$\frac{1}{4}(9 + x) + 10 = -3 + \frac{1}{2}x$

Solution

$\frac{1}{4}(9 + x) + 10 = -3 + \frac{1}{2}x$

Multiply both sides by 4*:*

$(9 + x) + 40 = -12 + 2x$

Open brackets:

$9 + x + 40 = -12 + 2x$

Add 12 to both sides of the equation:

$9 + x + 40 + 12 = -12 + 2x + 12$

Collect the like terms in both sides:

$(9 + 40 + 12) + x = (-12 + 12) + 2x$

Simplify:

$61 + x = 2x$

Subtract x from both sides of the equation:

$61 + x - x = 2x - x$

Simplify:

$61 = x$

Swap the left-hand and the right-hand sides of the equation:

$x = 61$

Answer: $x = 61$.

Question #3

Solve the following equation, giving answer as integer, surd or fraction in its simplest form

$$\frac{x}{x-4} - \frac{2}{x+3} = 1$$

Solution

Method 1

$$\frac{x}{x-4} - \frac{2}{x+3} = 1$$

Rewrite the first term in the left-hand side of the equation:

$$\frac{x-4+4}{x-4} - \frac{2}{x+3} = 1$$

$$\frac{x-4}{x-4} + \frac{4}{x-4} - \frac{2}{x+3} = 1$$

$$1 + \frac{4}{x-4} - \frac{2}{x+3} = 1$$

Subtract 1 from both sides of the equation:

$$1 + \frac{4}{x-4} - \frac{2}{x+3} - 1 = 1 - 1$$

Collect the like terms in both sides:

$$(1-1)+\frac{4}{x-4}-\frac{2}{x+3}=(1-1)$$

Simplify:

$$\frac{4}{x-4}-\frac{2}{x+3}=0$$

Add $\frac{2}{x+3}$ *to both sides of the equation:*

$$\frac{4}{x-4}=\frac{2}{x+3}$$

Use the property of proportions (it follows from $\frac{a}{b}=\frac{c}{d}$ *that* $ad=bc$*. In other words, the cross products of the two equal ratios will be equal):*

$$4(x+3)=2(x-4)$$

Open brackets

$$4x+12=2x-8$$

Subtract 12 from both sides of the equation:

$$4x+12-12=2x-8-12$$

Simplify:

$$4x=2x-20$$

Subtract $2x$ *from both sides of the equation:*

$$4x-2x=2x-20-2x$$

Collect the like terms in both sides:

$$(4x-2x)=(2x-2x)-20$$

Simplify:

$$2x=-20$$

Divide both sides by 2:

$$x=-10$$

If $x=-10$*, then the denominators will be non-zero:*

$$x-4=-10-4=-14\neq 0,\ x+3=-10+3=-7\neq 0.$$

Method 2

$$\frac{x}{x-4}-\frac{2}{x+3}=1$$

Reduce the left-hand side of the equation to the common denominator:

$$\frac{x(x+3)-2(x-4)}{(x-4)(x+3)}=1$$

Open brackets in the numerator and denominator:

$$\frac{x^2+3x-2x+8}{x^2-x-12}=1$$

Multiply both sides by $x^2-x-12\neq 0$:

$$x^2+3x-2x+8=x^2-x-12$$

Subtract x^2 *from both sides of the equation:*

$$x^2+3x-2x+8-x^2=x^2-x-12-x^2$$

Collect the like terms in both sides:

$$(x^2-x^2)+(3x-2x)+8=(x^2-x^2)-x-12$$

Simplify:

$$x+8=-x-12$$

Add x *to both sides of the equation:*

$$x+8+x=-x-12+x$$

Collect the like terms in both sides:

$$(x+x)+8=(-x+x)-12$$

Simplify:

$$2x+8=-12$$

Subtract 8 *from both sides of the equation:*

$$2x+8-8=-12-8$$

Simplify:

$$2x=-20$$

Divide both sides by 2:

$$x=-10.$$

If $x = -10$*, then the denominators will be non-zero:*

$x - 4 = -10 - 4 = -14 \neq 0$, $x + 3 = -10 + 3 = -7 \neq 0$.

Answer: $x = -10$.