Question

if $x = \frac{1}{2} - \sqrt{3}$ then find the value of $x^3 - 2x^2 - 7x - 4$

Solution

If $x = \frac{1}{2} - \sqrt{3}$, then

$$x^3 - 2x^2 - 7x - 4 = \left(\frac{1}{2} - \sqrt{3}\right)^3 - 2\left(\frac{1}{2} - \sqrt{3}\right)^2 - 7\left(\frac{1}{2} - \sqrt{3}\right) - 4 =$$

$$= \left(\frac{1}{2}\right)^3 - 3\left(\frac{1}{2}\right)^2 \sqrt{3} + 3 \cdot \frac{1}{2} \cdot (\sqrt{3})^2 - (\sqrt{3})^3 - 2\left[\left(\frac{1}{2}\right)^2 - 2 \cdot \frac{1}{2} \cdot \sqrt{3} + (\sqrt{3})^2\right] -$$

$$-\frac{7}{2} + 7\sqrt{3} - 4 =$$

$$= \frac{1}{8} - 3 \cdot \frac{1}{4} \cdot \sqrt{3} + 3 \cdot \frac{1}{2} \cdot 3 - 3\sqrt{3} - 2\left[\frac{1}{4} - \sqrt{3} + 3\right] - \frac{7}{2} + 7\sqrt{3} - 4 =$$

$$= \frac{1}{8} - \frac{3}{4}\sqrt{3} + \frac{9}{2} - 3\sqrt{3} - \frac{2}{4} + 2\sqrt{3} - 6 - \frac{7}{2} + 7\sqrt{3} - 4 =$$

$$= \left(\frac{1}{8} + \frac{9}{2} - \frac{2}{4} - \frac{7}{2}\right) + \left(-\frac{3}{4} - 3 + 2 + 7\right)\sqrt{3} - 6 - 4 =$$

$$= \frac{1 + 9 \cdot 4 - 2 \cdot 2 - 7 \cdot 4}{8} + \left(-\frac{3}{4} + 6\right)\sqrt{3} - 10 =$$

$$= \frac{1 + 36 - 4 - 28}{8} + \frac{-3 + 6 \cdot 4}{4}\sqrt{3} - 10 =$$

$$= \frac{5}{8} + \frac{21}{4}\sqrt{3} - 10 =$$

$$= \frac{5}{8} - 10 + \frac{21}{4}\sqrt{3} = \frac{5 - 8 \cdot 10}{8} + \frac{21}{4}\sqrt{3} = -\frac{75}{8} + \frac{21}{4}\sqrt{3}.$$

Answer: $-\frac{75}{8} + \frac{21}{4}\sqrt{3}$.