Answer on Question #61735 – Math – Differential Equations

Question #1

If u = f(x,y) be a function of two independent variables x and y, then ∂u/∂y is equal to

a) lim∆x→0(f(x+∆x,y)−f(x,y))/∆x
b) lim∆y→0(f(x,y+∆y)−f(x,y))/∆y
c) limy→0(f(x,y+∆y)−f(x,y))/∆y
d) lim∆y→0(f(x+∆x,y)−f(x,y))/∆x

Solution

$\frac{\partial u}{\partial y} = \lim_{\Delta y \to 0} \frac{f(x,y+\Delta y)-f(x,y)}{\Delta y}$ according to the definition of the partial derivative.

Answer: b) $\lim_{\Delta y \to 0} \frac{f(x,y+\Delta y)-f(x,y)}{\Delta y}$.

Question #2

Find the total differential of the function:

u = x^2y−3y.

a)2x+(x^2−3)dy
b) 2xydx+x^2dy
c) 2xydx+(x^2−3)dy
d) 2xydx+(x^3−2)dy

Solution

Let x and y be independent variables. Then the total differential of the function

u = f(x, y) is calculated by the following formula:

$$du = df(x, y) = \frac{\partial f(x,y)}{\partial x}dx + \frac{\partial f(x,y)}{\partial y}dy,$$

where $\frac{\partial f(x,y)}{\partial x}$ and $\frac{\partial f(x,y)}{\partial y}$ are partial derivatives of the function f(x, y) with respect to x and y respectively.

We hold y fixed and allow x to vary, then

$$\frac{\partial f(x,y)}{\partial x} = \frac{\partial}{\partial x}(x^2y-3y) = \frac{\partial}{\partial x}(x^2y) - \frac{\partial}{\partial x}(3y) = y\frac{\partial}{\partial x}(x^2) - (3y)\frac{\partial}{\partial x}(1) =$$

$$= 2xy - 0 = 2xy.$$

We hold x fixed and allow y to vary, then

$\frac{\partial f(x,y)}{\partial y} = \frac{\partial}{\partial y}(x^2 y - 3y) = \frac{\partial}{\partial y}(x^2 y) - \frac{\partial}{\partial y}(3y) = x^2 \frac{\partial}{\partial y}(y) - 3\frac{\partial}{\partial y}(y) = x^2 - 3.$

Thus,

du = df(x, y) = $\frac{\partial f(x,y)}{\partial x}dx + \frac{\partial f(x,y)}{\partial y}dy$ = $2xy$dx + (x^2−3)dy.

Answer: c) $2xy$dx + (x^2 − 3)dy.

Question #3

The total differential du of a function u (x, y) = 0 is defined as …

a) ∂u/∂xdx+∂u/∂xdy=0
b) ∂u/∂xdy+∂u/∂xdy=0
c) ∂u/∂xdx+∂u/∂ydx=0
d)∂u/∂xdx+∂u/∂ydy=0

Solution

Let x and y be independent variables. Then the total differential of the function

u = f(x, y) is calculated by the following formula:

$$\text{du} = \text{df}(x, y) = \frac{\partial f(x,y)}{\partial x}dx + \frac{\partial f(x,y)}{\partial y}dy,$$

where $\frac{\partial f(x,y)}{\partial x}$ and $\frac{\partial f(x,y)}{\partial y}$ are partial derivatives of the function f(x, y) with respect to x and y respectively.

Thus, the correct answer is

$$\frac{\partial f(x,y)}{\partial x}dx + \frac{\partial f(x,y)}{\partial y}dy = 0.$$

Answer: d) $\frac{\partial f(x,y)}{\partial x}dx + \frac{\partial f(x,y)}{\partial y}dy = 0.$