

Answer on Question #61725 – Math – Algebra

Question

What is the Vertex Formula, and how does it work?

Solution

A vertex formula is a formula used to calculate the coordinates of the vertex of the parabola

$$y = ax^2 + bx + c, \quad (1)$$

x coordinate is calculated by

$$x = -\frac{b}{2a} \quad (2)$$

and y coordinate is calculated by

$$y = \frac{4ac - b^2}{4a} \quad (3)$$

or by substituting x found in (2) into the formula (1).

Example.

The parabola

$$y = 2x^2 - 4x + 5$$

has x coordinate of the vertex given by the following formula:

$$x = -\frac{-4}{2 \cdot 2} = 1.$$

Let's find y coordinate of the vertex using formula (2):

$$y = \frac{4 \cdot 2 \cdot 5 - (-4)^2}{4 \cdot 2} = \frac{40 - 16}{8} = \frac{24}{8} = 3$$

Let's also calculate y coordinate of the vertex by substituting $x=1$ into

$$y = 2x^2 - 4x + 5:$$

$$y = 2 \cdot 1^2 - 4 \cdot 1 + 5 = 2 - 4 + 5 = 3$$

We can see two ways of calculations gave the same answer.

Thus, coordinates of the vertex of the parabola are $(x; y)=(1; 3)$.

Remark.

Similarly to (1), (2), (3), a vertex formula is also used to calculate the coordinates of the vertex of the parabola

$$x = ay^2 + by + c, \quad (4)$$

y coordinate is calculated by

$$y = -\frac{b}{2a} \quad (5)$$

and x coordinate is calculated by

$$x = \frac{4ac - b^2}{4a} \quad (6)$$

or by substituting y found in (5) into the formula (4).

Parabolas $y^2 = 2px$, $y^2 = -2px$, $x^2 = 2py$, $x^2 = -2py$ have the vertex at $(0; 0)$.

Parabolas $(y - b)^2 = 2p(x - a)$, $(y - b)^2 = -2p(x - a)$,

$(x - a)^2 = 2p(y - b)$, $(x - a)^2 = -2p(y - b)$ have the vertex at $(a; b)$.