Question

1. A bag contains 6 blue balls,4 red balls and 3 yellow balls.
   If a ball is picked at random from the bag, find the probability that it is

   red

   red and blue

   blue or yellow.

Solution

The number of red balls is 4 and the total number of balls is 13, then the probability of picking a red ball is:

$$P = \frac{4}{13}$$

Since the ball cannot be both red and blue, the probability of picking red and blue ball is

$$P = 0$$

The number of blue, yellow balls is 9 and the total number of balls is 13, then the probability of picking a blue or yellow ball is

$$P = \frac{9}{13}$$

Answer: $\frac{4}{13}$; 0; $\frac{9}{13}$.

Question

2. If two balls are picked at random from the bag (A) with replacement (B) without replacement
   Find the probability that:

   both blue

   the first is blue and the second is black

   the same colour

   different colours

Solution

(A) with replacement.

The number of blue balls is 6 and the total number of balls is 13, then the probability of picking a blue ball is

$$P = \frac{6}{13}$$

Then the probability of picking two blue balls with replacement is

$$P = \frac{6}{13} \cdot \frac{6}{13} = \frac{36}{169}$$

Since the bag does not contain black balls, the probability of picking the first blue and the second black ball is

$$P = 0$$

Let A_1 be the event of picking two blue balls. Then probability of A_1 is

$$P(A_1) = \frac{36}{169}$$

Let A_2 be the event of picking two red balls. Then probability of A_2 is

$$P(A_2) = \frac{4}{13} \cdot \frac{4}{13} = \frac{16}{169}$$

Let A_3 be the event of picking two yellow balls. Then probability of A_3 is

$$P(A_3) = \frac{3}{13} \cdot \frac{3}{13} = \frac{9}{169}$$

Let A be the event of picking two balls of the same color. Then the probability of A is

$$P(A) = P(A_1) + P(A_2) + P(A_3) = \frac{36}{169} + \frac{16}{169} + \frac{9}{169} = \frac{61}{169}$$

Let B be the event of picking two balls of different colors. Then the probability of this event is

$$P(B) = 1 - P(A) = 1 - \frac{61}{169} = \frac{108}{169}$$

(B) without replacement

The number of blue balls is 6 and the total number of balls is 13, then the probability of picking a blue ball is

$$P = \frac{6}{13}$$

After this there are 5 blue balls and totally 12 balls in the bag.
Then the probability of picking two blue balls with replacement is

$$P = \frac{6}{13} \cdot \frac{5}{12} = \frac{30}{156} = \frac{5}{26}$$

Since the bag does not contain black balls, the probability of picking the first blue and the second black ball is

$$P = 0$$

Let A_1 be the event of picking two blue balls. Then probability of A_1 is

$$P(A_1) = \frac{5}{26}$$

Let A_2 be the event of picking two red balls. Then probability of A_2 is

$$P(A_2) = \frac{4}{13} \cdot \frac{3}{12} = \frac{12}{156} = \frac{1}{13}$$

Let A_3 be the event of picking two yellow balls. Then probability of A_3 is

$$P(A_3) = \frac{3}{13} \cdot \frac{2}{12} = \frac{6}{156} = \frac{1}{26}$$

Let A be the event of picking two balls of the same color. Then the probability of picking two balls of the same color is

$$P(A) = P(A_1) + P(A_2) + P(A_3) = \frac{5}{26} + \frac{1}{13} + \frac{1}{26} = \frac{5}{26} + \frac{2}{26} + \frac{1}{26} = \frac{8}{26} = \frac{4}{13}$$

Let B be the event of picking two balls of different colors. Then the probability of this event is

$$P(B) = 1 - P(A) = 1 - \frac{4}{13} = \frac{9}{13}$$

Answer: (A) $\frac{36}{169}$; 0; $\frac{61}{169}$; $\frac{108}{169}$; **(B)** $\frac{5}{26}$; 0; $\frac{4}{13}$; $\frac{9}{13}$.