Answer on Question #61597–Math –Linear Algebra

Question

Find the quadratic polynomial whose graph goes through the points (−2,10), (0,6) and (1,10).

Solution

Consider the quadratic polynomial

$$y(x)=ax^2+bx+c \quad \textbf{(1)}$$

and substitute the given coordinates of points:

$y(-2)=10$ then $a\cdot(-2)^2+b\cdot(-2)+c=10$, that is, $4a-2b+c=10$;

$y(0)=6$ then $a\cdot 0^2+b\cdot 0+c=6$, so $c=6$;

$y(1)=10$ then $a\cdot 1^2+b\cdot 1+c=10$, that is, $a+b+c=10$.

So we get the system of three equations which we can be solved using elimination:

$$c=6,$$
$$4a-2b+c=10,$$
$$a+b+c=10.$$

Since c = 6, we can substitute it into the other two equations to get:

$$4a-2b+6=10 \text{ and } a+b+6=10.$$

Simplifying to

$$4a-2b=4, \quad \textbf{(2)}$$
$$a+b=4. \quad \textbf{(3)}$$

Dividing (2) by 2

$$2a-b=2. \quad \textbf{(4)}$$

It follows from (3) that

$$b=4-a. \quad \textbf{(5)}$$

Using (5) and substituting for b into (4) one gets

$$2a-(4-a)=2;$$
$$2a-4+a=2,$$
$$3a=6,$$
$$a=2.$$

Then $b=4-a=4-2=2$.

Thus, a=2, b=2 and c=6.

Substituting those values into (1) one gets the general equation of the quadratic polynomial that passes through the three given points:

$$y(x)=2x^2+2x+6.$$

Answer: $y(x)=2x^2+2x+6$.