Answer on Question #61376 – Math – Calculus

Question

7 Given

$y = \frac{2t^5 - 5}{t^2}$,

find $\frac{dy}{dx}$ by using the first principle.

−t−2+8t−3

6t+7t−3

t2+5t−3

6t2+10t−3

Solution

$$y = \frac{2t^5 - 5}{t^2} = y = 2t^3 - 5t^{-2}$$

$$y(t+h) = 2(t+h)^3 - 5(t+h)^{-2} = 2(t^3 + 3h^2t + 3t^2h + h^3) - 5\frac{1}{(t^2 + 2ht + h^2)}$$

$$\begin{aligned}\frac{dy}{dt} = \lim_{h\to 0}\frac{y(t+h) - y(t)}{h} &= \lim_{h\to 0}\frac{\left(2(t^3 + 3h^2t + 3t^2h + h^3) - 5\frac{1}{(t^2 + 2ht + h^2)}\right) - (2t^3 - 5t^{-2})}{h} \\ &= \lim_{h\to 0}\frac{2h^3 + 6th^2 + 6ht^2}{h} - 5\lim_{h\to 0}\frac{1}{h}\left(\frac{1}{(t^2 + 2ht + h^2)} - \frac{1}{t^2}\right) \\ &= \lim_{h\to 0}(2h^2 + 6th + 6t^2) \\ &- 5\lim_{h\to 0}\frac{1}{h}\cdot\frac{t^2 - t^2 - 2ht - h^2}{t^2(t^2 + 2ht + h^2)} = (2\cdot 0^2 + 6t\cdot 0 + 6t^2) + 5\lim_{h\to 0}\frac{2t + h}{t^2(t^2 + 2ht + h^2)} \\ &= 6t^2 + 5\frac{2t + 0}{t^2(t^2 + 2\cdot 0\cdot t + 0^2)} = 6t^2 + \frac{5\cdot 2t}{t^4} = 6t^2 + 10t^{-3}.\end{aligned}$$

Answer: $6t^2 + 10t^{-3}$.

Question

8 Given $y(x) = x^4 - 4x^3 + 3x^2 - 5x$, evaluate $\frac{d^4y}{dx^4}$

30

42

24

22

Solution

$\frac{dy}{dx} = \frac{d}{dx}(x^4 - 4x^3 + 3x^2 - 5x) = 4x^{4-1} - 4 \cdot 3x^{3-1} + 3 \cdot 2x^{2-1} - 5x^{1-1} = 4x^3 - 12x^2 + 6x - 5,$

$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}(4x^3 - 12x^2 + 6x - 5) = 4 \cdot 3x^{3-1} - 12 \cdot 2x^{2-1} + 6x^{1-1} - 0 = 12x^2 - 24x + 6,$

$\frac{d^3y}{dx^3} = \frac{d}{dx}\left(\frac{d^2y}{dx^2}\right) = \frac{d}{dx}(12x^2 - 24x + 6) = 12 \cdot 2x^{2-1} - 24x^{1-1} + 0 = 24x - 24,$

$$\frac{d^4y}{dx^4} = \frac{d}{dx}\left(\frac{d^3y}{dx^3}\right) = \frac{d}{dx}(24x - 24) = 24x^{1-1} - 0 = 24$$

Answer: 24.