

Answer on Question #61375 – Math – Calculus

Question

5 Evaluate the limit $\lim_{x \rightarrow \infty} \left(\frac{6e^{4x} - e^{-2x}}{8e^{4x} - e^{-2x} + 3e^{-x}} \right)$

a) 3/4

b) 1/4

c) 1/2

d) 3/5

Solution

$$\begin{aligned} \lim_{x \rightarrow \infty} \left(\frac{6e^{4x} - e^{-2x}}{8e^{4x} - e^{-2x} + 3e^{-x}} \right) &= \\ &= \lim_{x \rightarrow \infty} \left(\frac{(6e^{4x} - e^{-2x}) : e^{4x}}{(8e^{4x} - e^{-2x} + 3e^{-x}) : e^{4x}} \right) = \lim_{x \rightarrow \infty} \left(\frac{6 - \frac{e^{-2x}}{e^{4x}}}{8 - \frac{e^{-2x}}{e^{4x}} + \frac{3e^{-x}}{e^{4x}}} \right) \\ &= \lim_{x \rightarrow \infty} \left(\frac{6 - e^{-6x}}{8 - e^{-6x} + 3e^{-5x}} \right) = \frac{\lim_{x \rightarrow \infty} (6 - e^{-6x})}{\lim_{x \rightarrow \infty} (8 - e^{-6x} + 3e^{-5x})} = \frac{6 - 0}{8 - 0 + 3 \cdot 0} = \frac{6}{8} = \frac{3}{4}. \end{aligned}$$

Answer: a) $\frac{3}{4}$.

Question

6) Find the derivative $f(x) = 2x^2 - 16x + 35$ by using first principle

a) $x+16$

b) $4x-16$

c) $3x-5$

d) $2x-8$

Solution

$$f(x+h) = 2(x+h)^2 - 16(x+h) + 35 = 2h^2 + 4xh + 2x^2 - 16h - 16x + 35.$$

The derivative is

$$\begin{aligned} \frac{df}{dx} &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{(2h^2 + 4xh + 2x^2 - 16h - 16x + 35) - (2x^2 - 16x + 35)}{h} \\ &= \lim_{h \rightarrow 0} \frac{2h^2 + 4xh - 16h}{h} = \lim_{h \rightarrow 0} (2h + 4x - 16) = 2 \cdot 0 + 4x - 16 = 4x - 16. \end{aligned}$$

Answer: b) $4x - 16$.