

Answer on Question #61284 - Math - Analytic Geometry

Question

A straight line slides along axes (oblique) (of x and y , and the difference of the intercepts is always proportional to the area it encloses. Show that the line always passes through a fixed point.

Solution

$$\frac{x}{a} + \frac{y}{b} = 1 \quad (1)$$

Difference of the intercepts is $a - b$, area is $\frac{ab}{2}$.

It is given that

$$\frac{a-b}{ab} = \text{constant} = \frac{1}{k}, \quad (2)$$

where k is fixed.

Multiplying both sides of (2) by k

$$k \frac{a-b}{ab} = 1$$

Then

$$\frac{k \cdot a}{a \cdot b} + \frac{-k \cdot b}{a \cdot b} = 1$$

$\frac{-k}{a} + \frac{k}{b} = 1 \Rightarrow (-k, k)$ is a point of the straight line according to the equation (1).

Thus, the line always passes through the fixed point $(-k, k)$.