Answer on Question #61279 – Math – Calculus

Question

For the infinite geometric sequence (x to base n) whose first four terms are 1.5, 3.6, 8.64 & 20.736,

1) find the values of the first term a and the common ratio r and write down a recurrence system for this sequence;

2) write a closed form for this sequence;

3) calculate the 7th term of sequence;

4) how many terms of sequence are less than 30000?

Solution

1) The value of the first term:

$$a_1 = 1.5$$

The common ratio r :

$$r = \frac{3.6}{1.5} = 2.4$$

A recurrence system:

$$a_n = a_{n-1} \times 2.4, \text{ where } a_1 = 1.5$$

2) A closed form for this sequence:

$$a_n = a_1 \times r^{n-1} = 1.5 \times 2.4^{n-1}$$

3) The 7th term of sequence:

$$a_7 = a_1 \times r^6 = 1.5 \times 2.4^6 = 286.654464$$

4) The value of the k[th] term is less than 30000:

$$a_k \leq 30000$$

Using the formula found in the second part

$$a_1 \times r^{k-1} \leq 30000$$
$$1.5 \times 2.4^{k-1} \leq 30000$$
$$2.4^{k-1} \leq 20000$$
$$k - 1 \leq \log_{2.4} 20000$$
$$k - 1 \leq 11.3$$
$$k = 12$$

Let`s check the answer:

$$a_{12} = 22825.2$$
$$a_{13} = 54780.5$$

So the a_{12} is the last term less than 30000.

Answer:

1) $a_1 = 1.5, \quad r = 2.4, \quad a_n = a_{n-1} \times 2.4$;

2) $a_n = 1.5 \times 2.4^{n-1}$;

3) $a_7 = 286.654464$;

4) 12.