Answer on Question #61276 – Math – Linear Algebra

Question

Solve the systems using the Gaussian elimination method.

x+2y+z=4

2x-y-3z=2

x-8y-9z=-8

5x+5y =14

Solution

I'll convert the system to its upper triangular form.

1) Multiply the first row by -2 and add it to the second row:

$$-2(x+2y+z)+(2x-y-3z)=-2x-4y-2z+2x-y-3z=-5y-5z;$$

$$-2\cdot 4+2=-6.$$

The result is written in the second row:

$$\begin{array}{rrrrr} x & +2y & +z & = & 4 \\ & -5y & -5z & = & -6 \\ x & -8y & -9z & = & -8 \\ 5x & +5y & & = & 14 \end{array}$$

2) Multiply the first row by -1 and add it to the third row:

$$-(x+2y+z)+(x-8y-9z)=-x-2y-z+x-8y-9z=-10y-10z;$$

$$-4+(-8)=-12.$$

The result is written in the third row:

$$\begin{array}{rrrrr} x & +2y & +z & = & 4 \\ & -5y & -5z & = & -6 \\ & -10y & -10z & = & -12 \\ 5x & +5y & & = & 14 \end{array}$$

3) Multiply the first row by -5 and add it to the fourth row:

$$-5(x+2y+z)+(5x+5y)=-5x-10y-5z+5x+5y=-5y-5z;$$

$$-5\cdot 4+14=-6.$$

The result is written in the fourth row:

$$\begin{array}{rrrrr} x & +2y & +z & = & 4 \\ & -5y & -5z & = & -6 \\ & -10y & -10z & = & -12 \\ & -5y & -5z & = & -6 \end{array}$$

4) Divide the third row by 2:

$$\begin{array}{rrrrr} x & +2y & +z & = & 4 \\ & -5y & -5z & = & -6 \\ & -5y & -5z & = & -6 \\ & -5y & -5z & = & -6 \end{array}$$

5) Hence we have the system:

$$\begin{array}{rrrrr} x & +2y & +z & = & 4 \\ & -5y & -5z & = & -6 \end{array}$$

6) Divide the third row by -5:

$$\begin{array}{rrrrr} x & +2y & +z & = & 4 \\ & y & +z & = & 6/5 \end{array}$$

7) Subtract the second row from the first row:

$$\begin{array}{rrrrr} x & +y & & = & 4-6/5 \\ & y & +z & = & 6/5 \end{array}$$

obtain

$$\begin{array}{rrrrr} x & +y & & = & 14/5 \\ & y & +z & = & 6/5 \end{array}$$

This system has infinite number of solutions, because the number of equations $(k = 2)$ is less than the number of variables $(n = 3)$.

If *y* is an independent variable, then it follows from the first equation of the system that $x = 14/5 - y$, it follows from the second equation of the system that $z = \frac{6}{5} - y$. Thus, $(x, y, z) = \left(\frac{14}{5} - s, s, \frac{6}{5} - s\right)$, where $s \in \mathbb{R}$.

If *z* is an independent variable, then it follows from the second equation of the system that

$$y = \frac{6}{5} - z.$$

Substituting it into the first equation of the system

$$x + y = \tfrac{14}{5};$$

$$x+\frac{6}{5}-z=\frac{14}{5};$$

$$x=\frac{14}{5}-\frac{6}{5}+z;$$

$$x=\frac{8}{5}+z.$$

Thus, $(x,y,z)=\left(t+\frac{8}{5},\frac{6}{5}-t,t\right)$, where $t\in\mathbb{R}$.

If *x* is an independent variable, then it follows from the first equation of the system that

$$y=\frac{14}{5}-x.$$

Substituting it into the second equation of the system

$$y+z=\frac{6}{5}$$

$$\frac{14}{5}-x+z=\frac{6}{5};$$

$$z=\frac{6}{5}-\frac{14}{5}+x=x-\frac{8}{5};$$

Thus, $(x,y,z)=\left(u,\frac{14}{5}-u,u-\frac{8}{5}\right)$, where $u\in\mathbb{R}$.

Answer: $(x,y,z)=\left(u,\frac{14}{5}-u,u-\frac{8}{5}\right)$, where $u\in\mathbb{R}$,

or

$(x,y,z)=\left(\frac{14}{5}-s,s,\frac{6}{5}-s\right)$, where $s\in\mathbb{R}$,

or

$(x,y,z)=\left(t+\frac{8}{5},\frac{6}{5}-t,t\right)$, where $t\in\mathbb{R}$.