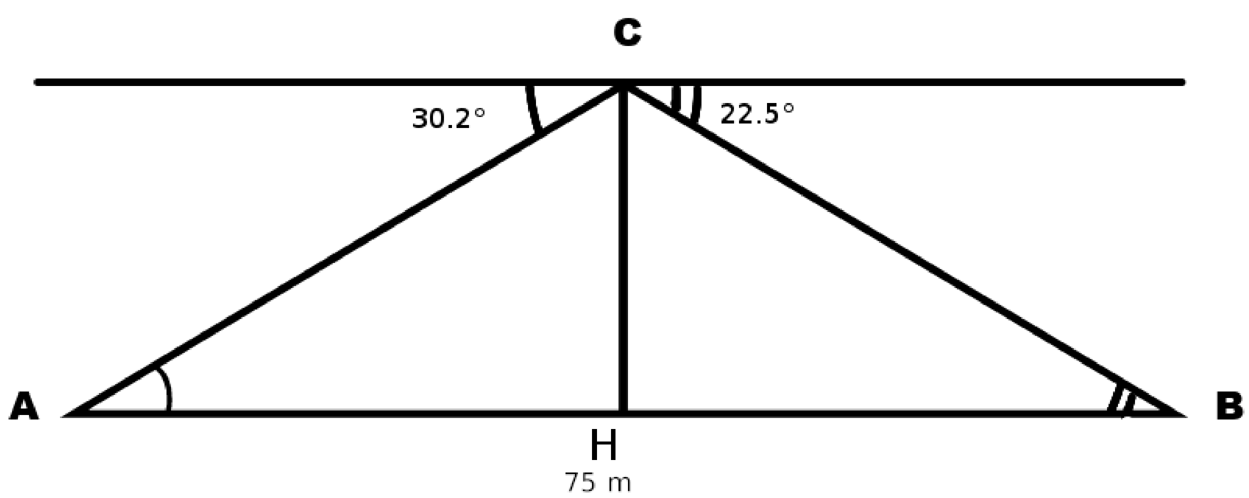
Answer on Question #61236 – Math – Trigonometry

Question

Points A and B are on the same horizontal line with the foot of a hill and the angles of depression of these points from the top of the hill are 30.2° and 22.5°, respectively. If the distance between A and B is 75 m, what is the height of the hill?



Solution

Method 1

The straight line, where lies point C, and the straight line, where lies points A and B, are parallel, because first straight is line of horizon and second is foot of hill. In this case lines AC and BC are secants of two parallel straight lines. Then angles of depression of points A and B from the top of the hill and angles A and B are equal in measure as alternate interior angles. Hence A = 30.2° and B = 22.5°.

Now using the theorem about the sum of angles of a triangle, find the measure of angle C:

$C = 180° - A - B = 180° - 30.2° - 22.5° = 127.3°$.

Therefore triangle ABC is obtuse.

Now apply the theorem of sines:

$\frac{AB}{sin(C)} = \frac{BC}{sin(A)} = \frac{AC}{sin(B)} = 2R,$

hence

$BC = \frac{AB \times sin(A)}{sin(C)},$

$AC = \frac{AB \times sin(B)}{sin(C)},$

$2R = \frac{AB}{sin(C)}$.

Deduce the formula for the height of the triangle through two sides and the radius of the circumscribed circle:

$\mathrm{H} = \frac{2S}{a}$,

where S – area of a triangle, a – the basis of height

$$S = \frac{abc}{4R}$$

where R – radius of circumscribed triangle

From the above formulae it follows:

$$H = \frac{2S}{a} = \frac{2 \times abc}{a \times 4R} = \frac{bc}{2R}$$

Therefore,

$$CH = \frac{BC \times AC}{2R} = \frac{AB \times \sin(A)}{\sin(C)} \times \frac{AB \times \sin(B)}{\sin(C)} \times \frac{\sin(C)}{AB} = \frac{AB \times \sin(A) \times \sin(B)}{\sin(C)},$$

$$CH = \frac{47.42 \times 35.8}{94.28} \approx 18.1(m).$$

Answer:

The height of the hill is 18.1 m.

Method 2

$AH = x$, $BH = y$. Then $AB = AH + BH = x + y = 75$. By definition of the tangent, $\tan(A) = \tan(30.2^{\circ}) = \frac{CH}{AH} = \frac{CH}{x}$, $\tan(B) = \tan(22.5^{\circ}) = \frac{CH}{BH} = \frac{CH}{y}$, hence

$CH = xtan\ (30.2^{\circ})$ and $CH = ytan\ (22.5^{\circ})$.

Equating two formulae $CH = xtan(30.2^{\circ}) = ytan(22.5^{\circ})$.

Obtain the system of equations

$$\begin{cases} x + y = 75, \\ x\ tan(30.2^{\circ}) = ytan\ (22.5^{\circ}). \end{cases}$$

It follows from the first equation that $x = 75 - y$, substituting into the second equation obtain $(75 - y)\ tan(30.2^{\circ}) = ytan\ (22.5^{\circ})$.

Hence

$$y = \frac{75tan(30.2^{\circ})}{tan(22.5^{\circ}) + tan(30.2^{\circ})}$$

and

$$CH = ytan(22.5^{\circ}) = \frac{75tan(30.2^{\circ}) \times tan(22.5^{\circ})}{tan(22.5^{\circ}) + \tan(30.2^{\circ})} = 18.1\ (\mathrm{m})$$

Answer:

The height of the hill is 18.1 m.