Answer on Question #61215 – Math – Statistics and Probability

Question

A) The weight of the adult females has a mean around 60 kg and a standard deviation of 20kg. If the sample of 16 adult females was chosen.

i) What is the probability that the average weight of these 16 randomly selected females will be below 60kg?

ii) What is the probability that the average weight of these 16 randomly selected females will exceed 75kg?

iii) What is the probability that the average weight of these 16 randomly selected females will be between 65kg and 75 kg?

Solution

Suppose that a simple random sample of size $n = 16$ is drawn from a population with mean $\mu = 60$ and standard deviation $\sigma = 20$. The sample mean $\bar{x}$ is normally distributed, with mean $\mu_{\bar{x}} = \mu = 60$ and standard deviation $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{20}{\sqrt{16}} = 5$.

i) The probability that the average weight of these 16 randomly selected females will be below 60kg is

$$P(\bar{x} < 60) = P\left(Z < \frac{60-\mu_{\bar{x}}}{\sigma_{\bar{x}}}\right) = P\left(Z < \frac{60-\mu}{\frac{\sigma}{\sqrt{n}}}\right) = P\left(Z < \frac{60-60}{\frac{20}{\sqrt{16}}}\right) = P(Z < 0) = 0.5.$$

The random variable Z has the standard normal distribution (with mean $\mu_Z = 0$ and standard deviation $\sigma_Z = 1$).

ii) The probability that the average weight of these 16 randomly selected females will exceed 75kg is

$$P(\bar{x} > 75) = P\left(Z > \frac{75-\mu_{\bar{x}}}{\sigma_{\bar{x}}}\right) = P\left(Z > \frac{75-\mu}{\frac{\sigma}{\sqrt{n}}}\right) = P\left(Z > \frac{75-60}{\frac{20}{\sqrt{16}}}\right) = P(Z > 3) = P(Z < -3) = 0.00135.$$

The random variable Z has the standard normal distribution (with mean $\mu_Z = 0$ and standard deviation $\sigma_Z = 1$).

iii) The probability that the average weight of these 16 randomly selected females will be between 65kg and 75 kg is

$$P(65 < \bar{x} < 75) = P\left(\frac{65 - \mu_{\bar{x}}}{\sigma_{\bar{x}}} < Z < \frac{75 - \mu_{\bar{x}}}{\sigma_{\bar{x}}}\right) = P\left(\frac{65 - \mu}{\frac{\sigma}{\sqrt{n}}} < Z < \frac{75 - \mu}{\frac{\sigma}{\sqrt{n}}}\right) =$$

$$= P\left(\frac{65 - 60}{\frac{20}{\sqrt{16}}} < Z < \frac{75 - 60}{\frac{20}{\sqrt{16}}}\right) = P(1 < Z < 3) =$$

$$= P(Z < 3) - P(Z < 1) = 0.99865 - 0.84134 = 0.15731.$$

Answer: **i)** 0.5; **ii)** 0.00135; **iii)** 0.15731.

Question

B) In the city of Hongkong 70% of the people prefer a swamp candidate for a mayoral position. Supposed 30 people from Hongkong were sampled.

i) What is the mean of the sampling distribution of P (sample proportion)?

ii) What is the standard error of p?

iii) What is the probability that 80% of this sample will prefer a candidate from swamp?

Solution

i) The mean of the sampling distribution of p (sample proportion) is

$$\hat{p} = 0.7.$$

ii) The standard error of p is

$$SE = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{0.7(1-0.7)}{30}} = 0.083666.$$

iii)

Check The random variable $X = \text{number od successes}$ has the binomial distribution

with mean $\mu_X = np = 30 \cdot 0.7 = 21$ and

standard deviation $\sigma_X = \sqrt{np(1-p)} = \sqrt{30 \cdot 0.7 \cdot (1-0.7)} = \sqrt{30 \cdot 0.7 \cdot 0.3} = \sqrt{\frac{630}{100}} = \frac{3}{10}\sqrt{70}.$

Check $np = 30 \cdot 0.7 = 21 > 10$ and $np(1-p) = 30 \cdot 0.7 \cdot (1-0.7) = 30 \cdot 0.7 \cdot 0.3 = 6.3 < 10.$

Calculate the following probability using the binomial probability distribution:

$$P(\hat{p} = 0.8) = P\left(\frac{X}{n} = 0.8\right) = P(X = 0.8n) = P(X = 0.8 \cdot 30) = P(X = 24) = \binom{30}{24} p^{24}(1-p)^{30-24} =$$

$$= \frac{30!}{24!(30-24)!} 0.7^{24} 0.3^{6} = \frac{30 \cdot 29 \cdot 28 \cdot 27 \cdot 26 \cdot 25}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2} 0.7^{24} 0.3^{6} = 0.08923.$$

Answer: i) 0.7; **ii)** 0.083666; **iii)** 0.08923.