Answer on Question #61005 – Math – Calculus

Question

(a) Let H(t) be the total weight of all the lobsters, in kg, after t months.
Find a formula for H(t), expressing it in simplest form, Since the number of lobsters in the tank in months is expressed as N(t)= 6000/(t^2+25) and the mean weight of lobsters in months is expressed as W(t)=1.5ln (t/4+1)

(b) draw this function and find its first derivative, H'(t),

(c) Consider the graph of H'(t). By choosing a suitable option, determine the rate at which the total weight of all the lobsters is changing after 2 months and after 10 months. Explain the meaning of your answers with reference to their magnitude and sign.

Solution

(a)

$$H(t) = N(t) \cdot W(t)$$

$$H(t) = \frac{6000}{t^2 + 25} \cdot 1.5 \ln(\frac{t}{4} + 1) = \frac{9000}{t^2 + 25} \ln(\frac{t}{4} + 1)$$

(b)

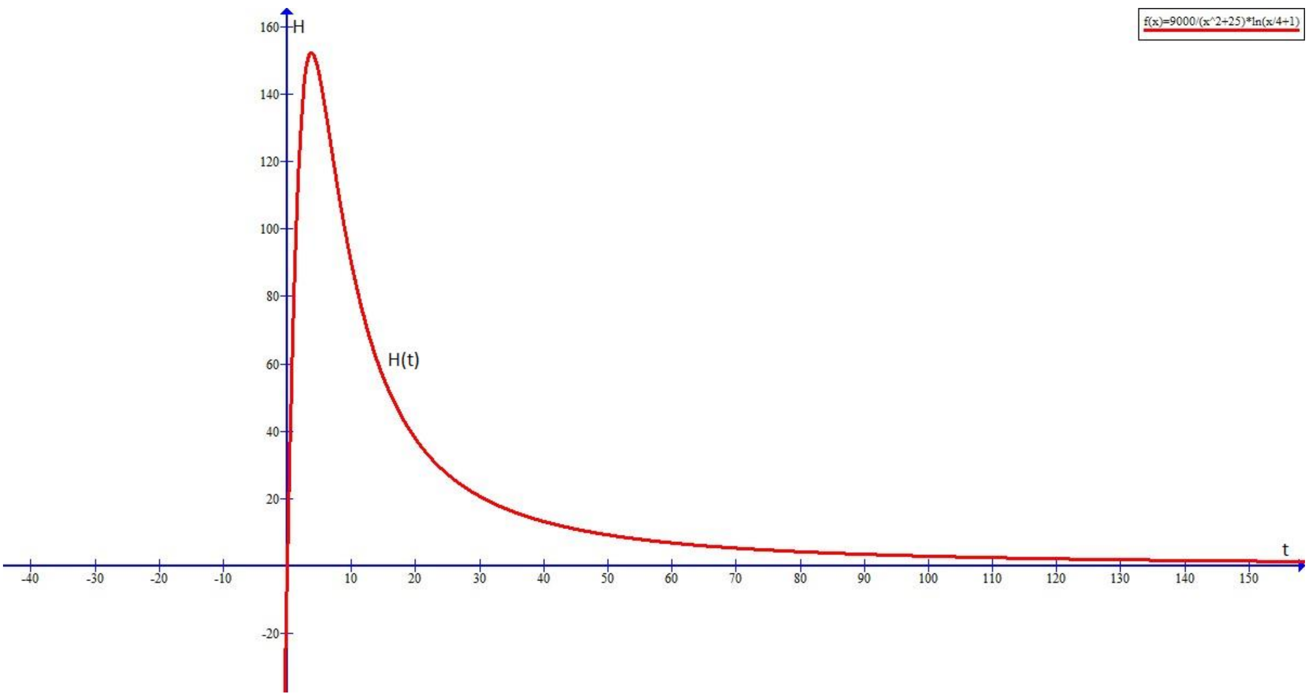

Fig. 1 Plot of function $H(t)$

$$H'(t) = \left(\frac{9000}{t^2+25}\ln\left(\frac{t}{4}+1\right)\right)' = \frac{-9000\cdot 2t}{(t^2+25)^2}\cdot \ln(\frac{t}{4}+1) + \frac{9000}{t^2+25}\cdot\frac{1\cdot\frac{1}{4}}{\frac{t}{4}+1} =$$

$$= \frac{-18000t}{(t^2+25)^2}\cdot\ln(\frac{t}{4}+1) + \frac{9000}{(t^2+25)}\cdot\frac{1}{t+4} = \frac{-18000t}{(t^2+25)^2}\cdot\ln(\frac{t}{4}+1) + \frac{9000}{t^3+4t^2+25t+100}$$

(c)

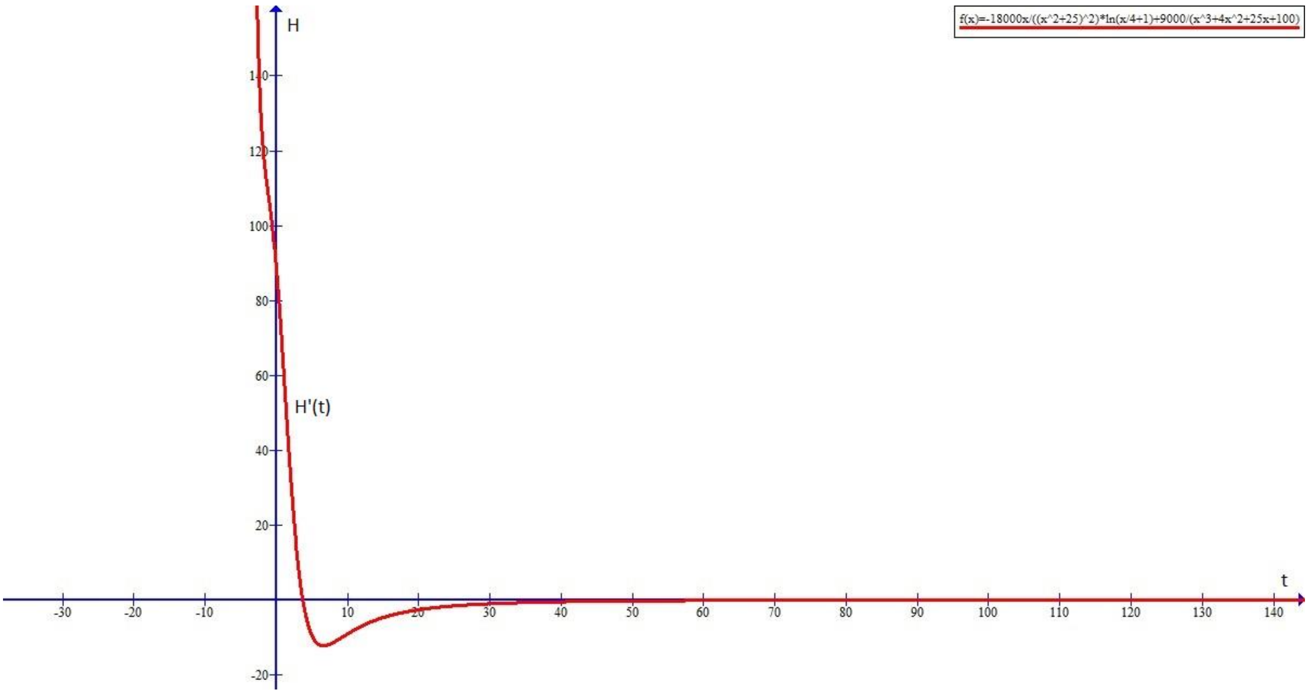

Fig. 2 Plot of $H'(t)$

The rate at which the total weight of all the lobsters is changing after 2 months is given by

$$H'(2) = \frac{-18000\cdot 2}{(2^2+25)^2}\cdot\ln(\frac{2}{4}+1) + \frac{9000}{2^3+4\cdot 2^2+25\cdot 2+100} = 34.36$$

(here magnitude is 34.36, the sign is positive, hence the total weight of all the lobsters is increasing after 2 months).

The rate at which the total weight of all the lobsters is changing after 10 months is given by

$$H'(10) = \frac{-18000\cdot 10}{(10^2+25)^2}\cdot\ln(\frac{10}{4}+1) + \frac{9000}{10^3+4\cdot 10^2+25\cdot 10+100} = -9.28$$

(here magnitude is 9.28, the sign is negative, hence the total weight of all the lobsters is decreasing after 10 months).