Answer on Question #60957 – Math – Discrete Mathematics

Question

How many integers between 1 and 99 inclusive are divisible by either 2 or 3 or both?

Solution

Let's determine sets A and B in the following way:

A ='integers between 1 and 99 inclusive, which are divisible by 2'; $A = \{a_1, a_2, \dots, a_n\}$;

B ='integers between 1 and 99 inclusive, which are divisible by 3'; $B = \{b_1, b_2, \dots, b_m\}$.

Thus,

A ∪ B = 'integers between 1 and 99 inclusive, which are divisible by either 2 or 3 or both';

A ∩ B ='integers between 1 and 99, which are divisible by both 2 and 3, thus, they are divisible by 6'; A ∩ B= $\{c_1, c_2, \dots, c_k\}$.

Let's find the number |A| of elements in the set A:

If $a_1 = 2,\ d = 2,\ a_n = 98$, then $a_n = a_1 + (n-1)d = 2 + 2(n-1) = 98$, hence

$n - 1 = \frac{98-2}{2}$, that is, $n = 1 + \frac{96}{2}$, $|\mathrm{A}| = n = 49$.

Let's find the number |B| of elements in the set B:

If $b_1 = 3, e = 3, b_m = 99$, then $b_m = b_1 + (m-1)e = 3 + 3(m-1) = 99$, hence

$m - 1 = \frac{99-3}{3}$, that is, $m = 1 + \frac{96}{3}$, $|\mathrm{B}| = m = 33$.

Let's find the number |A ∩ B| of elements in the set A ∩ B:

If $c_1 = 6, f = 6, c_k = 96$, then $c_k = c_1 + (k-1)f = 6 + 6(k-1) = 96$, hence

$k - 1 = \frac{96-6}{6}$, that is, $k = 1 + \frac{90}{6}$, $|\mathrm{A} \cap \mathrm{B}| = k = 16$.

Thus,

$|\mathrm{A}| = 49$; $|\mathrm{B}| = 33$; $|\mathrm{A} \cap \mathrm{B}| = 16$.

Let's find the number |A ∪ B| of elements in the set A ∪ B:

$|\mathrm{A} \cup \mathrm{B}| = |\mathrm{A}| + |\mathrm{B}| - |\mathrm{A} \cap \mathrm{B}|$;

$|\mathrm{A} \cup \mathrm{B}| = 49 + 33 - 16 = 66$.

Answer: 66.