Answer on Question #60952 – Math – Discrete Mathematics

Question

Prove that $1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$

Proof

We will use mathematical induction to prove this formula.

Basis: $n = 1$ (the least positive integer). The statement is

$$1^2 = \frac{1*2*3}{6}.$$

Obviously it's true.

Inductive step:

Assuming the statement is true for $n = k$:

$$1^2 + 2^2 + \ldots + k^2 = \frac{k(k+1)(2k+1)}{6}.$$

We must prove that the statement is true for $n = k + 1$:

$$1^2 + 2^2 + \ldots + k^2 + (k+1)^2 = \frac{(k+1)(k+2)(2k+3)}{6}.$$

Consider

$$(1^2 + 2^2 + \ldots + k^2) + (k+1)^2 = \frac{k(k+1)(2k+1)}{6} + (k+1)^2 =$$

$$= \frac{k(k+1)(2k+1) + 6(k+1)^2}{6} = \frac{(k+1)(2k^2 + k + 6k + 6)}{6} =$$

$$= \frac{(k+1)(2k^2 + 7k + 6)}{6} = \frac{(k+1)(k+2)(2k+3)}{6}$$

Therefore, by the principle of mathematical induction, the given statement is true for every positive integer *n*.