Answer on Question #60947 – Math – Analytic Geometry

Question

A line can either lie on a plane, lie parallel to it or intersect it.

a) Determine, if there is one, the point of intersection between the line given by the equation: $\frac{x-5}{2}=\frac{y-1}{-1}=\frac{z-15}{4}$ and the plane given by the equation:

$[x,y,z]=[-2,-7,5]+s[2,6,3]+t[1,4,-1]$.

b) Determine the angle between the line and the plane.

c) Give the equation of a plane and three lines, one of which is parallel to the plane, one of which lies on the plane, and one of which intersects the plane.

Solution

a) Let us rewrite the equation of the given plane in the coordinate form. It is well known that equation $[x,y,z]=[x_0,y_0,z_0]+s[a_1,a_2,a_3]+t[b_1,b_2,b_3]$ is equivalent to the next equation: $\begin{vmatrix} x-x_0 & y-y_0 & z-z_0 \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}=0$. So in our case we have the next equation: $\begin{vmatrix} x+2 & y+7 & z-5 \\ 2 & 6 & 3 \\ 1 & 4 & -1 \end{vmatrix}=0 \Leftrightarrow -18x+5y+2z-11=0$. So the direction vector of the given line is $\bar{v}=(2,-1,4)$, and the normal vector to the given plane is $\bar{n}=(-18,5,2)$. Since

$2\cdot(-18)+(-1)\cdot 5+4\cdot 2=-33\neq 0$ then the given line intersects the given plane.

To determine the point of intersection we must solve the following linear system: $\begin{cases} -18x+5y+2z-11=0 \\ \frac{x-5}{2}=\frac{y-1}{-1} \\ \frac{y-1}{-1}=\frac{z-15}{4} \end{cases} \Leftrightarrow \begin{cases} -18x+5y+2z-11=0 \\ x=7-2y \\ z=19-4y \end{cases} \Leftrightarrow \begin{cases} 36y+5y-8y=99 \\ x=7-2y \\ z=19-4y \end{cases} \Leftrightarrow$

$\Leftrightarrow \begin{cases} y=3 \\ x=1 \\ z=7 \end{cases}$. So the point of intersection is $(1,3,7)$.

b) We shall use the next formula: $\sin\varphi=\frac{|\bar{v}\cdot\bar{n}|}{|\bar{v}|\cdot|\bar{n}|}=\frac{|2\cdot(-18)+(-1)\cdot 5+4\cdot 2|}{\sqrt{2^2+(-1)^2+4^2}\cdot\sqrt{(-18)^2+5^2+2^2}}=\frac{33}{\sqrt{7413}}$. So

$\varphi=arcsin\frac{33}{\sqrt{7413}}$.

c) Let us consider the plane $z=0$.

The line $\frac{x}{1}=\frac{y}{0}=\frac{z-1}{0}$ is parallel to this plane because its direction vector $(1,0,0)$ is orthogonal to the normal vector $(0,0,1)$ of the plane and the point $(0,0,1)$ belongs to the line but does not belong to the plane.

The line $\frac{x}{1} = \frac{y}{1} = \frac{z}{0}$ obviously lies on the plane $z = 0$ because its direction vector $(1, 1, 0)$ is orthogonal to the normal vector $(0, 0, 1)$ of the plane and the point $(0, 0, 0)$ belongs to the line, and belongs to the plane.

The line $x = y = z$ intersects this plane because its direction vector $(1, 1, 1)$ is not orthogonal to the normal vector $(0, 0, 1)$ of the plane.

Answer:

a) $(1, 3, 7)$;

b) $arcsin \frac{33}{\sqrt{7413}}$;

c) $z = 0$; $\frac{x}{1} = \frac{y}{0} = \frac{z-1}{0}, \frac{x}{1} = \frac{y}{1} = \frac{z}{0}, x = y = z.$