Answer on Question #60913 – Math – Calculus

Question

A) Solve the following simultaneous inequalities
8x - 3≤ 1/4 [x + 16]< x + 10

B) Differentiate the following functions
y = x^3+ 〖6x〗 ^(-4)- 〖4x〗 ^(1/2)+ 10

y = (〖6x〗 ^3+ 〖4x〗 ^2+3)(〖2x〗 ^2- 〖4x〗 ^(-2)+5)

Solution

A)

Case 1. If $[x+16]=x+16$, then

$8x-3\le\frac{1}{4}[x+16]<x+10\to$

$8x-3\le\frac{x+16}{4}<x+10\to$

$32x-12\le x+16<4x+40\to$

$\begin{cases}32x-12\le x+16,\\ x+16<4x+40,\end{cases}\to$

$\begin{cases}32x-x\le 12+16,\\ 16-40<4x-x,\end{cases}\to$

$\begin{cases}31x\le 28,\\ -24<3x,\end{cases}\to$

$\begin{cases}x\le\dfrac{28}{31},\\ x>-8.\end{cases}$

Answer: $(-8,\frac{28}{31}]$.

Case 2. If $[x+16]=max\{m\in\mathbb{Z}|m\le x+16\}$, then

$8x-3\le\frac{1}{4}[x+16]<x+10\to\ \ 32x-12\le[x+16]<4x+40\ \to$

$$\to 32x-12\le[x]+16<4x+40\ \to$$
$$32x-28\le[x]<4x+24$$

Consider

$$[x] < 4x + 24 \Leftrightarrow \left[\begin{array}{c} x < 4x + 24 \; if \; 4x + 24 \; is \; integer \\ x < 4x + 25 \; if \; 4x + 24 \; is \; not \; integer \end{array}\right. \Leftrightarrow$$

$$\left[\begin{array}{c} -24 < 3x \; if \; 4x + 24 = k, \; k \; is \; integer \\ -25 < 3x \; if \; 4x + 24 \neq k, \; k \; is \; not \; integer \end{array}\right. \Leftrightarrow$$

$$\left[\begin{array}{l} x > -8 \; if \; x = \frac{k}{4} - 6, \; k \; is \; integer \\ x > -\frac{25}{3} \; if \; x \neq \frac{k}{4} - 6, \; k \; is \; integer \end{array}\right.$$

If $x = -8$, then $8 \cdot (-8) - 3 \leq \frac{1}{4}[-8 + 16] < -8 + 10$ is false, hence $x = -8$ is not a solution. If $x = -\frac{25}{3}$, then $8 \cdot \left(-\frac{25}{3}\right) - 3 \leq \frac{1}{4}\left[-\frac{25}{3} + 16\right] < -\frac{25}{3} + 10$ is false, therefore, $x = -\frac{25}{3}$ is not a solution. Check any number between $-\frac{25}{3}$ and -8, for example, $x = -\frac{81}{10}$: $8 \cdot \left(-\frac{81}{10}\right) - 3 \leq \frac{1}{4}\left[-\frac{81}{10} + 16\right] < -\frac{81}{10} + 10$,
$-64.8 \leq \frac{7}{4} < 1.9$ is true.

Consider

$$[x] \geq 32x - 28 \Leftrightarrow \left[\begin{array}{c} x \geq 32x - 28 \; if \; 32x - 28 \; is \; integer \\ x \geq 32x - 27 \; if \; 32x - 28 \; is \; not \; integer \end{array}\right. \Leftrightarrow$$

$$\left[\begin{array}{l} 31x \leq 28 \; if \; 32x - 28 = m, \; m \; is \; integer \\ 31x \leq 27 \; if \; 32x - 28 \neq m, \; m \; is \; integer \end{array}\right. \Leftrightarrow$$

$$\left[\begin{array}{l} x \leq \frac{28}{31} \; if \; x = \frac{m}{32} + \frac{7}{8}, m \; is \; integer \\ x \leq \frac{27}{31} \; if \; x \neq \frac{m}{32} + \frac{7}{8}, m \; is \; integer \end{array}\right.$$

If $x = \frac{28}{31}$, then $8 \cdot \left(\frac{28}{31}\right) - 3 \leq \frac{1}{4}\left[\frac{28}{31} + 16\right] < \frac{28}{31} + 10$ is wrong, hence $x = \frac{28}{31}$ is not a solution. If $x = \frac{27}{31}$, then $8 \cdot \left(\frac{27}{31}\right) - 3 \leq \frac{1}{4}\left[\frac{27}{31} + 16\right] < \frac{27}{31} + 10$ is true, hence $x = \frac{27}{31}$ is a solution. Check any number between $\frac{27}{31}$ and $\frac{28}{31}$, for example, $x = \frac{9}{10}$:
$8 \cdot \left(\frac{9}{10}\right) - 3 \leq \frac{1}{4}\left[\frac{9}{10} + 16\right] < \frac{9}{10} + 10$, which is false.

Answer: $\left(-\frac{25}{3}, -8\right) \cup (-8, \frac{27}{31}]$.

B)

Case 1. Assuming $[6x] = 6x$, $[4x] = 4x$, $[2x] = 2x$.

If

$y = x^3 + (6x)^{-4} - (4x)^{\frac{1}{2}} + 10$, then

$$y' = 3x^2 - \frac{4}{1296}x^{-5} - x^{-\frac{1}{2}} = 3x^2 - \frac{1}{324x^5} - \frac{1}{\sqrt{x}}.$$

If

$y = ((6x)^3 + (4x)^2 + 3)((2x)^2 - (4x)^{-2} + 5) =$

$= 864x^5 + 64x^4 + 1080x^3 + 92x^2 - \frac{27}{2}x - \frac{3}{16x^2} + 14$, then

$y' = 4320x^4 + 256x^3 + 3240x^2 + 184x + \frac{3}{8x^3} - \frac{27}{2}$.

Case 2.

Assuming $[6x] = max\{m \in \mathbb{Z} | m \leq 6x\}$, $[4x] = max\{k \in \mathbb{Z} | k \leq 4x\}$,

$[2x] = max\{l \in \mathbb{Z} | l \leq 2x\}$.

If

$y = x^3 + ([6x])^{-4} - ([4x])^{\frac{1}{2}} + 10$, then

$y' = 3x^2$, if $x \neq \frac{k}{6}, x \neq \frac{l}{4}$, k is integer, l is integer,

y' does not exist if $x = \frac{k}{6}$ or $x = \frac{l}{4}$, k is integer, l is integer.

If $y = (([6x])^3 + ([4x])^2 + 3)(([2x])^2 - ([4x])^{-2} + 5)$, then

$y' = 0$, if $x \neq \frac{k}{6}, x \neq \frac{l}{4}$, k is integer, l is integer;

y' does not exist if $x = \frac{k}{6}$ or $x = \frac{l}{4}$, k is integer, l is integer.